Trigonometry

1) The unit of angle measure is degree and it is denoted by ^{0}.

2) The unit of angle measure is radian and it is denoted by ^{c}.

3) π radian = 180 degree i.e. $\pi^C = 180^0$,

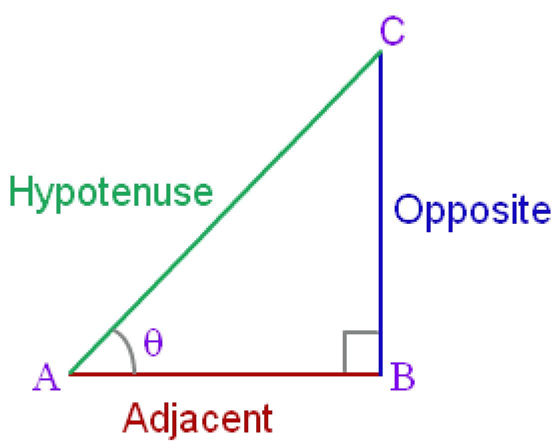
$1\text{radian} = \left(\frac{180}{\pi}\right)\text{degree}$ i.e. $1^C = \left(\frac{180}{\pi}\right)^0$,

4) $1\text{degree} = \left(\frac{\pi}{180}\right)$ radian i.e. $1^0 = \left(\frac{\pi}{180}\right)^C$,

Trigonometric Ratios

The ratios which relate the sides of a right angle to its angles are called trigonometric ratios.
There are six trigonometric ratios of an angle θ. They are $\sin\theta, \cos\theta, \tan\theta, \cot\theta, \sec\theta$ and $\operatorname{cosec}\theta$

Consider ΔABC is a right angled triangle as shown in the figure where $\angle A = \theta$. AB is adjacent side, BC is opposite side and AC is hypotenuse, then



1) $\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC}$, 2) $\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AB}{AC}$, 3) $\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{BC}{AB}$,

4) $\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{AC}{BC}$, 5) $\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{AC}{AB}$, 6) $\cot\theta = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{AB}{BC}$.

Trigonometric functions of standard angles

Angle / Ratio	0^0	$30^0 = (\pi/6)^c$	$45^0 = (\pi/4)^c$	$60^0 = (\pi/3)^c$	$90^0 = (\pi/2)^c$	$180^0 = \pi$	$270^0 = (3\pi/2)^c$	$360^0 = (2\pi)^c$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	∞	1
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞

Signs of trigonometric functions

Quadrants / Trigonometric ratios	I	I I	III	IV
sin	+	+	-	-
cos	+	-	-	+
tan	+	-	+	-

From above table, we say that
In I quadrant – all are positive.
In II quadrant – sine and cosec are positive
In III quadrant – tan and cot are positive
In IV quadrant – cos and sec are positive.

Trigonometric Ratios of Compound and Allied Angles

Compound Angle [BTE 2016] – An angle obtained by algebraic sum or difference of two or more angles is called a compound angle.
e.g. If A,B,C--- are angles, then $A+B,\ A-B, A+B+C,\ A-B-C---$ are compound angles.

Allied Angles – If the sum or difference of the measures of two angles is either zero or is an integral multiple of 90^0 or $n\dfrac{\pi}{2}$ where $n \in I$, then these angles are called allied angles.
e.g. If is the measure of a given angle, then its allied angles are of the form 0, $\dfrac{\pi}{2}+\theta, \dfrac{3\pi}{2}-\theta, 2\pi+\theta, 2\pi-\theta$ etc.

Trigonometric functions of addition and subtraction

1) $\sin(A+B) = \sin A\cos B + \sin B\cos A,$ 2) $\sin(A-B) = \sin A\cos B - \sin B\cos A,$

3) $\cos(A+B) = \cos A\cos B - \sin A\sin B,$ 4) $\cos(A-B) = \cos A\cos B + \sin A\sin B,$

5) $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A.\tan B},$ 6) $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A.\tan B},$

7) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B,$ 8) $\cos(A+B)\cos(A-B) = \cos^2 A - \cos^2 B.$

Trigonometric functions of allied angles

Angle / Ratio	$-\theta$	$\frac{\pi}{2}-\theta$	$\frac{\pi}{2}+\theta$	$\pi-\theta$	$\pi+\theta$	$\frac{3\pi}{2}-\theta$	$\frac{3\pi}{2}+\theta$	$2\pi-\theta$	$2\pi+\theta$
sin	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$
cos	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$	$\cos\theta$
tan	$-\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$
cosec	$-\text{cosec}\theta$	$\sec\theta$	$\sec\theta$	$\text{cosec}\theta$	$-\text{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\theta$	$\text{cosec}\theta$
sec	$\sec\theta$	$\text{cosec}\theta$	$-\text{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\theta$	$\text{cosec}\theta$	$\sec\theta$	$\sec\theta$
cot	$-\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\tan\theta$

Examples:
Without using the calculator, find the value of

1) $\cos(75)^0$.**[BTE 2017]** $\left[\cos(75)^0 = \dfrac{\sqrt{3}-1}{2\sqrt{2}}\right]$ 2) $\tan(75)^0$ **[BTE2012]** $\left[\tan(75)^0 = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right]$

3) $\sin 210^0$ [-1/2] 4) $\sec 3660^0$ [2]

5) $\sin 150^0 + \cos 300^0 - \tan 315^0 + \sec^2 3660^0$ [6]
$[\sin 150^0 = 1/2,\ \cos 300^0 = 1/2,\ \tan 315^0 = -1, \sec 3660^0 = 2]$

6) $\dfrac{\sec^2 135^0}{\cos(-240^0) - 2\sin 930^0}$ [4]

Examples for Tutorial

Without using the calculator, prove that

1) $\sin 420^0 \cos 390^0 + \cos(-300^0)\sin(-330^0) = 1$ [**BTE2016]**

$[\sin 420^0 = \sqrt{3}/2,\ \ \cos 390^0 = \sqrt{3}/2,\ \ \cos(-300^0) = 1/2,\ \ \sin(-330^0) = 1/2]$

2) $\sin 150^0 - \tan 315^0 + \cos 300^0 + \sec^2 360^0 = 3$[**BTE2015]**

$[\sin 150^0 = 1/2,\ \tan 315^0 = -1,\ \ \cos 300^0 = 1/2,\ \ \sec 360^0 = 1]$

Examples:

Prove that

1) $\sin(n+1)A.\sin(n+2)A + \cos(n+1)A.\cos(n+2)A = \cos A$

2) $\dfrac{\sin(A-B)}{\cos(A-B)} = \dfrac{\cot A + \cot B}{1 + \cot A \cot B}$

3) If $\tan x = \dfrac{5}{6}$ and $\tan y = \dfrac{1}{11}$, prove that $x + y = \dfrac{\pi}{4}$**.** **[BTE2014]**

4) In any ΔABC, prove that $\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$. **[BTE2014]**

$[A + B = 180^0 - C,\ \tan(180^0 - C) = -\tan C]$

5) Prove that $\dfrac{1-\tan 2\theta.\tan\theta}{1+\tan 2\theta.\tan\theta} = \dfrac{\cos 3\theta}{\cos\theta}$. **[BTE2013]**

6) If $\sin A = \dfrac{4}{5}, \dfrac{\pi}{2} < A < \pi$ and $\cos B = \dfrac{5}{13},\ \dfrac{3\pi}{2} < B < 2\pi$, find a) $\sin(A+B)$, b) $\cos(A-B)$.

$[\sin(A+B) = \dfrac{56}{65},\ \ \cos(A-B) = -\dfrac{63}{65}]$

Examples for Tutorial

1) $\sin(45^0 + A).\cos(45^0 - B) + \cos(45^0 + A).\sin(45^0 - B) = \cos(A - B)$

2) $\dfrac{\sin(A-B)}{\sin A \sin B} + \dfrac{\sin(B-C)}{\sin B \sin C} + \dfrac{\sin(C-A)}{\sin C \sin A} = 0$

3) $\dfrac{\sin(A+B)}{\sin(A-B)} = \dfrac{\tan A + \tan B}{\tan A - \tan B}$

4) If $\tan A = \dfrac{1}{2}$ and $\tan B = \dfrac{1}{3}$, find $\tan(A+B)$.**[BTE2016]** $[\tan(A+B) = 1]$

5) Show that $\tan 3A - \tan 2A - \tan A = \tan 3A.\tan 2A.\tan A$. $[3A = 2A + A]$

6) Prove that $1 + \tan\theta.\tan 2\theta = \sec 2\theta$. **[BTE2015]**

7) If $\sin\alpha = -\dfrac{5}{13}, \cos\beta = -\dfrac{7}{25}$ and α, β lies in the third quadrant, find $\sin(\alpha - \beta)$. **[BTE2013]**

$$\begin{bmatrix} \cos\alpha = -\dfrac{12}{13}\ (\alpha \text{ is in } III \text{ quadrant, } \cos\alpha \text{ is negative.}),\ \sin\beta = -\dfrac{24}{25}(\beta \text{ is in } III \text{ quadrant, } \sin\beta \text{ is negative.}) \\ \sin(\alpha-\beta) = -\dfrac{253}{325} \end{bmatrix}$$

Trigonometric Ratios of Multiple and Sub-multiple Angles

Multiple Angles – Angles of the form $2\theta, 3\theta, 4\theta ---$ are integral multiples of θ. They are called multiple angles.

Sub-multiple Angles – Angles of the form $\frac{\theta}{2}, \frac{3\theta}{2}, ---$ are called sub-multiple angles of θ.

Trigonometric functions of double and triple angles:

1) $\sin 2\theta = 2\sin\theta\cos\theta$, 2) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$,

3) $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$, 4) $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$, 5) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

6) $1-\cos 2\theta = 2\sin^2\theta$, $\therefore \sin^2\theta = \dfrac{1-\cos 2\theta}{2}$, 7) $1+\cos 2\theta = 2\cos^2\theta$, $\therefore \cos^2\theta = \dfrac{1+\cos 2\theta}{2}$,

8) $1+\sin 2\theta = (\cos\theta + \sin\theta)^2$, 9) $1-\sin 2\theta = (\cos\theta - \sin\theta)^2$,

10) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, 11) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, 12) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^3\theta}$.

Trigonometric functions of half angles:

1) $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$, 2) $\cos\theta = \cos^2(\theta/2) - \sin^2(\theta/2) = 1 - 2\sin^2(\theta/2) = 2\cos^2(\theta/2) - 1$,

3) $\sin\theta = \dfrac{2\tan(\theta/2)}{1+\tan^2(\theta/2)}$, 4) $\cos\theta = \dfrac{1-\tan^2(\theta/2)}{1+\tan^2(\theta/2)}$, 5) $\tan\theta = \dfrac{2\tan(\theta/2)}{1-\tan^2(\theta/2)}$

6) $1-\cos\theta = 2\sin^2(\theta/2)$, 7) $1+\cos\theta = 2\cos^2(\theta/2)$,

8) $1+\sin\theta = \left[\cos(\theta/2) + \sin(\theta/2)\right]^2$, 9) $1-\sin\theta = \left[\cos(\theta/2) - \sin(\theta/2)\right]^2$.

Examples

1) If $\theta = 45^0$, verify that

   a) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ b) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

2) If $\cos\theta = -\dfrac{12}{13}$ and $\pi < \theta < \dfrac{3\pi}{2}$, find a) $\sin 2\theta$ $\left[\dfrac{120}{169}\right]$ b) $\cos 2\theta$ $\left[\dfrac{119}{169}\right]$

3) If $\sin A = \dfrac{1}{2}$, find $\sin 3A$. $[\sin 3A = 1]$ **[BTE2014]**

4) Prove that

   a) $\dfrac{\sin\theta + \sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$ b) $\dfrac{\sin 9\theta}{\sin 3\theta} - \dfrac{\cos 9\theta}{\cos 3\theta} = 2$

   c) $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

Examples for Tutorial

1) If $\theta = 60^0$, verify that

a) $\sin 2\theta = 2\sin\theta\cos\theta$ b) $\cos 2\theta = 2\cos^2\theta - 1$

2) If $\sin\theta = 0.6$, find $\sin 3\theta$. $[\sin 3\theta = 0.936]$

3) If $\sin\theta = 0.4$, find $\cos 3\theta$. $[\cos 3\theta = 0.3297]$

4) If $\sin A = 0.4$, find $\sin 3A$. $[\sin 3A = 0.944]$ **[BTE2014]**

5) Prove that a) $\dfrac{\sin 2\theta + \cos\theta}{1 + \sin\theta - \cos 2\theta} = \cot\theta$ b) $\dfrac{\sec 4\theta - 1}{\sec 2\theta - 1} = \dfrac{\tan 4\theta}{\tan\theta}$

c) $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$

Examples on half angle formulae

1) If $\theta = 45^0$, find the values of $\sin(\theta/2)$ and $\cos(\theta/2)$ without using the calculator.

[Use the formulae $1 - \cos\theta = 2\sin^2(\theta/2)$, $1 + \cos\theta = 2\cos^2(\theta/2)$

$\sin(\theta/2) = \dfrac{\sqrt{2 - \sqrt{2}}}{2}$, $\cos(\theta/2) = \dfrac{\sqrt{2 + \sqrt{2}}}{2}$]

2) Prove that a) $\dfrac{\sin(\theta/2) + \sin\theta}{1 + \cos(\theta/2) + \cos\theta} = \tan(\theta/2)$ b) $\tan\left(45^0 - \dfrac{\theta}{2}\right) = \sqrt{\dfrac{1 - \sin A}{1 + \sin A}}$

Examples for Tutorial

1) If $\tan(\alpha/2) = \sqrt{3}$, find $\cos\alpha$. $[\cos\alpha = \cos 120 = -1/2]$

2) If $\tan(A/2) = \dfrac{1}{\sqrt{3}}$, find $\sin A$ $[\sin A = \sqrt{3}/2]$

3) Prove that a) $\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan(\theta/2)$ b)

Factorization and Defactorization Formulae

Factorization – The process of conversion from sum/difference into product is called factorization.

Defactorization - The process of conversion from product of terms into sum/difference is called defactorization.

Factorization Formulae:

1) $\sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$, 2) $\sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$,

3) $\cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$,

4) $\cos C-\cos D=2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)=-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$.

Defactorization Formulae:

1) $2\sin A\cos B=\sin(A+B)+\sin(A-B)$, 2) $2\cos A\sin B=\sin(A+B)-\sin(A-B)$,

3) $2\cos A\cos B=\cos(A+B)+\cos(A-B)$, 4) $2\sin A\sin B=\cos(A-B)-\cos(A+B)$.

Examples

A) Express the following as a sum or difference of trigonometric functions.

1) $2\sin 4\theta\cos 2\theta$ $[2\sin 4\theta\cos 2\theta=\sin 6\theta+\sin 2\theta]$

2) $2\cos 117^0\ \sin 53^0$ $[2\cos 117^0\ \sin 53^0=\sin 170^0-\sin 64^0]$

B) Express the following as product of trigonometric functions:

1) $\sin 2\theta+\sin 4\theta$ $[2\sin 3\theta\cos\theta]$ 2) $\cos 70^0+\cos 20^0$ $[2\cos 45^0\cos 25^0]$

C) 1) If $2\cos 60^0.\cos 10^0=\cos A+\cos B$, then find A and B. **[BTE2013]** $[A=70, B=50]$

2) Prove that a) $\frac{\sin 4\theta-\sin 2\theta}{\cos 4\theta+\cos 2\theta}=\tan\theta$ b) $\frac{\cos 3A-2\cos 5A+\cos 7A}{\cos A-2\cos 3A+\cos 5A}=\cos 2A-\sin 2A.\tan 3A$

c) $\frac{\sin 19^0+\cos 11^0}{\cos 19^0-\sin 11^0}=\sqrt{3}$ $\left[\sin 19^0=\cos(90^0-19^0)=\cos 71^0,\quad \cos 19^0=\sin(90^0-19^0)=\sin 71^0\right]$

d) $\cos 10^0\cos 50^0\cos 70^0=\frac{\sqrt{3}}{8}$ e) $\sin 10^0\sin 30^0\sin 50^0\sin 70^0=\frac{1}{16}$

f) $(\cos A+\cos B)^2+(\sin A+\sin B)^2=4.\cos^2\left(\frac{A-B}{2}\right)$

Examples for Tutorial

A) Express the following as a sum or difference of trigonometric functions.

1) $2\cos 4\theta\cos 2\theta$ $[2\cos 4\theta\cos 2\theta=\cos 6\theta+\cos 2\theta]$

2) $\sin(\theta/4)\sin(3\theta/4)$ $\left\{\sin(\theta/4)\sin(3\theta/4)=\frac{1}{2}\left[\cos(\theta/2)-\cos\theta\right]\right\}$

B) Express the following as product of trigonometric functions:

1) $\sin 7\theta - \sin 5\theta$ $[2\cos 6\theta \sin\theta]$

2) $\cos\frac{\pi}{13} - \cos\frac{2\pi}{13}$ $\left[2\sin\frac{3\pi}{26}\sin\frac{\pi}{26}\right]$

C) 1) If $\sin 80^0 + \sin 50^0 = 2\sin\alpha.\cos\beta$, find α and β. $[\alpha = 65^0,\ \ \beta = 15^0]$

2) Prove that a) $\frac{\cos 2B - \cos 2A}{\sin 2A + \sin 2B} = \tan(A-B)$

b) $\frac{\sin A + 2\sin 2A + \sin 3A}{\cos A + 2\cos 2A + \cos 3A} = \tan 2A$

c) $\frac{\cos 11^0 + \sin 11^0}{\cos 11^0 - \sin 11^0} = \tan 56^0$

d) $\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}$ **[BTE2016]**

e) $8\sin 20^0 \sin 40^0 \cos 10^0 = \sqrt{3}$

f) $(\cos A - \cos B)^2 + (\sin A - \sin B)^2 = 4.\sin^2\left(\frac{A-B}{2}\right)$

Inverse Trigonometric Functions

Definition - If $\sin\theta = x$, then $\theta = \sin^{-1} x$ is inverse trigonometric function. It is read as sine inverse of x.

Similarly $\cos^{-1} x, \tan^{-1} x, \cot^{-1} x, \sec^{-1} x, \text{cosec}^{-1} x$ are inverse trigonometric functions.

Relation between inverse trigonometric ratios :

1) $\sin^{-1}(x) = \text{cosec}^{-1}(1/x)$, $\text{cosec}^{-1}(x) = \sin^{-1}(1/x)$,

$\cos^{-1}(x) = \sec^{-1}(1/x)$, $\sec^{-1}(x) = \cos^{-1}(1/x)$,

$\tan^{-1}(x) = \cot^{-1}(1/x)$, $\cot^{-1}(x) = \tan^{-1}(1/x)$.

2) $\sin^{-1}(-x) = -\sin^{-1}(x)$, $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$,

$\tan^{-1}(-x) = -\tan^{-1}(x)$, $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$,

$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}(x)$, $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$.

3) $\sin^{-1}(\sin x) = x$, $\cos^{-1}(\cos x) = x$, $\tan^{-1}(\tan x) = x$,

$\cot^{-1}(\cot x) = x$, $\text{cosec}^{-1}(\text{cosec} x) = x$, $\sec^{-1}(\sec x) = x$.

4) $\sin^{-1} x + \cos^{-1} x = \pi/2$, $\tan^{-1} x + \cot^{-1} x = \pi/2$, $\text{cosec}^{-1} x + \sec^{-1} x = \pi/2$.

5) If $x > 0, y > 0$ and $xy < 1$ then, $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$.

6) If $x > 0, y > 0$ and $xy > 1$ then, $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$.

7) If $x > 0, y > 0$ then, $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$.

Note – 1) $\sin^{-1} x \neq \frac{1}{\sin x}$

Examples : Prove that

1) $\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)=\cot^{-1}\left(\frac{9}{2}\right)$ **[BTE2017]**

2) $\cot^{-1}\left(\frac{6}{5}\right)+\tan^{-1}\left(\frac{1}{11}\right)=\sec^{-1}\left(\sqrt{2}\right)$. **[BTE 2017]**

3) $\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right)=\sin^{-1}\left(\frac{77}{85}\right)$ **[BTE 2015, 2016]**

4) $\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{27}{11}\right)$ **[BTE 2015,17]**

Examples for Tutorial

1) $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\frac{\pi}{4}$ **[BTE 2017]**

2) $\sin^{-1}\left(\frac{3}{5}\right)-\sin^{-1}\left(\frac{8}{17}\right)=\cos^{-1}\left(\frac{84}{85}\right)$ **[BTE 2017]**

3) $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$ **[BTE 2016]**

4) $\cos^{-1}\left(\frac{4}{5}\right)-\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{63}{65}\right)$ **[BTE 2017]**

5) $\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$. **[BTE 2017]**

6) $\sin^{-1}\left(\frac{3}{5}\right)-\sin^{-1}\left(\frac{8}{17}\right)=\cos^{-1}\left(\frac{84}{85}\right)$. **[BTE 2017]**

Principal Value of Inverse Function

Definition – The smallest numerical value, either positive or negative, of an inverse trigonometric function is called as principal value of the function.

e.g. We know $\sin 30^0=\frac{1}{2}, \sin 150^0=\frac{1}{2}, \sin 390^0=\frac{1}{2}---$

$\therefore 30^0=\sin^{-1}\left(\frac{1}{2}\right), \quad 150^0=\sin^{-1}\left(\frac{1}{2}\right), \quad 390^0=\sin^{-1}\left(\frac{1}{2}\right)---$

$\therefore \sin^{-1}\left(\frac{1}{2}\right)$ have many values $30^0, 150^0, 390^0, ---$

$\therefore$ Principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is 30^0.

Examples: Find the principal value of the following:

1) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $\left(\frac{\pi}{4}\right)$ 2) $\sec\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$ $\left(\frac{2}{\sqrt{3}}\right)$

3) $\cos\left[\frac{\pi}{2}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ **[BTE2012]** $\left(-\frac{1}{2}\right)$

Examples for Tutorial

1) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ $\left(\frac{2\pi}{3}\right)$ 2) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ $\left(-\frac{\pi}{6}\right)$ 3) $\tan^{-1}(-1)$ $\left(-\frac{\pi}{4}\right)$

Syllabus for Mid Semester Examination - Oct. 2018

Unit I – Logarithm, Partial Fraction, Complex Numbers

Unit II – Determinant, Matrices