# Straight Line

**Slope of a line**

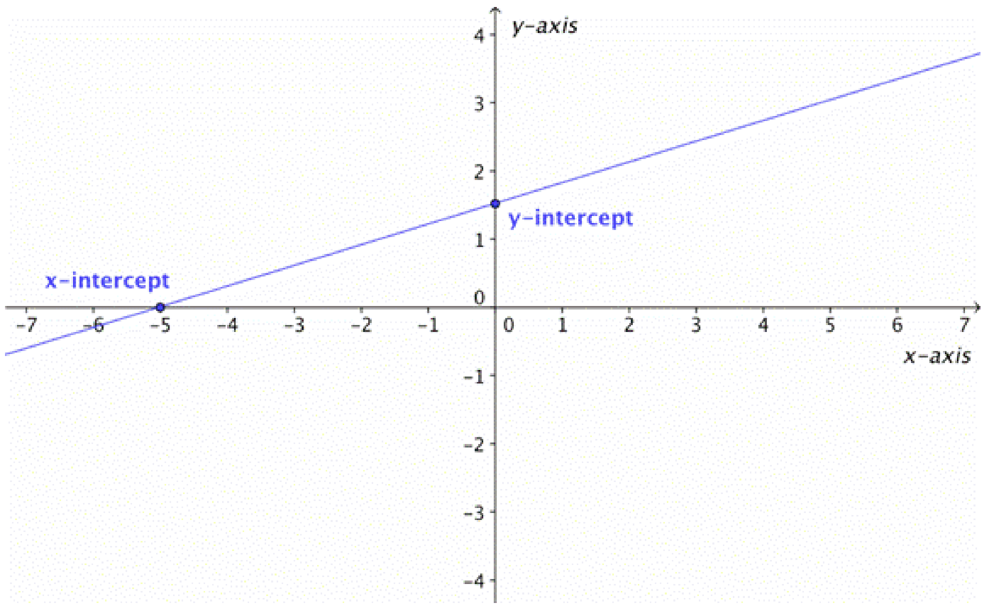
**Definition:** The slope or gradient of a line is defined as the tangent ratio of its inclination, provided that the line is not parallel to y-axis.

If a line makes an angle of measure $\theta$ with x-axis, then $\tan\theta$ is called the slope of a line and it is denoted by m.

$$\therefore m = \tan\theta$$

**Intercept**

The intercepts of a line are the points where the line intercepts, or crosses, the horizontal and vertical axes.




The point where the line crosses the *x*-axis is called the $x-\text{intercept}$. The point where the line crosses the *y*-axis is called $y-\text{intercept}$.

**Note:** 1) If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points on the line then slope of AB is denoted by m and is given by,

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

2) The general equation of a line is $ax + by + c = 0$ .

$$\text{Slope} = m = -\frac{a}{b} = -\frac{\text{Co-efficient of } x}{\text{Co-efficient of } y}, \qquad x-\text{intercept} = -\frac{c}{a} = -\frac{\text{Constant}}{\text{Co-efficient of } x},$$

$$y-\text{intercept} = -\frac{c}{b} = -\frac{\text{Constant}}{\text{Co-efficient of } y}$$

The general equation of a line is $ax+by+c=0$.

$$\therefore c=-ax-by$$

Dividing by $\sqrt{a^2+b^2}$ on both sides,

$$\therefore\left(-\frac{a}{\sqrt{a^2+b^2}}\right)x+\left(-\frac{b}{\sqrt{a^2+b^2}}\right)y=-\frac{c}{\sqrt{a^2+b^2}}$$

This is of the form $x\cos\alpha+y\sin\alpha=p$, where

$$\cos\alpha=-\frac{a}{\sqrt{a^2+b^2}},\quad \sin\alpha=-\frac{b}{\sqrt{a^2+b^2}}\quad \therefore \tan\alpha=\frac{b}{a}$$

$$p=-\frac{c}{\sqrt{a^2+b^2}}$$

3) Equation of $x$- axis is $y=0$.

4) Equation of y-axis is $x=0$.

5) The slope of $x$-axis is zero.

6) The slope of $y-$axis is not defined $(\infty)$ . It has no slope.

7) The $x-$intercept of a line is found by putting $y=0$.

8) The $y-$intercept of a line is found by putting $x=0$.

9) If the co-ordinates of a point $A(a,0)$ then the $x-$co$-$ordinate of a point A is called $x$-intercept of a line.

$$\therefore x\text{-intercept}=a$$

10) If the co-ordinates of a point $B(0,b)$ then the $y-$co$-$ordinate of a point B is called $y$-intercept of a line.

$$\therefore y\text{-intercept}=b$$

11) Three points A,B and C are collinear if slope of AB is equal to slope of AC.

12) Two lines having slope $m_1$ and $m_2$ are **parallel** to each other if their slopes are equal i.e. $m_1 = m_2$.

13) Two lines having slope $m_1$ and $m_2$ are **perpendicular** to each other if the product of slopes is $-1$.

i.e. $m_1m_2=-1$

## Different forms of line:

**Slope-Point Form:** The equation of a line having slope m and passing through the point $(x_1,y_1)$ is given by $y-y_1=m(x-x_1)$

**Note:** If the line passes through the origin having slope m, then the equation of line is $y=mx$.

**Slope-intercept Form:** The equation of a line having slope m and y-intercept c is $y=mx+c$.

**Note:** The equation of a line having slope m and x-intercept $c'$ is $y=mx+mc'$.

**Two-point Form:** The equation of a line passing through the points $(x_1,y_1)$ ,and $(x_2,y_2)$ is given by,

$$\frac{y-y_1}{y_1-y_2}=\frac{x-x_1}{x_1-x_2}.$$

**Two-intercept Form or Double Intercept Form:** The equation of a line having x-intercept ‘a’ and y-intercept ‘b’ is given by,

$$\frac{x}{a}+\frac{y}{b}=1$$

**Normal Form of a line:** The equation of a line with length of normal from origin is p and inclination of angle is $\alpha$ is $x\cos\alpha+y\sin\alpha=p$.

**Examples on slope and intercept**

A) Find the slope of a line whose inclination is as follows:

1) $45^0$ $[m=1]$ 2) $150^0$ $\left[m=-\frac{1}{\sqrt{3}}\right]$ 3) $\frac{2\pi^c}{3}$ $\left[m=-\sqrt{3}\right]$

B) Find the slope of a line passing through the following points:

1) $(-1,-2)$ and $(-3,8)$ **[BTE2014]** $[m=-5]$ 2) $(1,-3)$ and $(-1,-1)$ $[m=-1]$

3) having intercepts of equal magnitude but opposite signs on the co-ordinate axes. $[m=1]$

C) Find the value of K if the slope of a line passing through the following points:

1) $(2,4)$ and $(5,K)$ is 5/3 [K = 9] 2) $(-1,-4)$ and $(2,K)$ is $-1$ $[K=-7]$

D) Find the slope and intercepts of the following lines:

1) $3x+4y=12$ $[m=-3/4,\ x-\text{intercept}=4,\ y-\text{intercept}=3]$

2) $\frac{2x}{3}+\frac{y}{4}=5$ $[m=-8/3,\ x-\text{intercept}=15/3,\ y-\text{intercept}=20]$

E) Using slopes, find the value of K if the following points are collinear.

1) $(7,K),(3,6),(-5,-2)$ $[K=10]$ 2) $(1,1),(-2,-11),(K,-3)$ $[K=0]$

**Examples on slope-point form of a line**

1) Find the equation of line passing through the point $(-3,2)$ and having slope $5/2$.**[BTE2016]** $[5x-2y+19=0]$

2) Find the equation of a line passing through the point (2,-3) and making an angle of $135^0$ with positive direction of X – axis. **[IoPE 2016]** Also, find its X and Y – intercepts.

$$\left\{\begin{array}{l} m=\tan 135^0=\tan(180-45)=-\tan 45=-1\ [\because \tan(180-\theta)=-\tan\theta, \tan 45^0=-1] \\ \text{Ans. } x+y+1=0,\quad x-\text{intercept}=-1,\quad y-\text{intercept}=-1 \end{array}\right\}$$

**Examples on slope-intercept form of a line**

1) Find the equation of a line with a slope 1/2 and y- intercept is 7. $[y=(1/2)x+7]$

2) Find the equation of a line which makes an angle of $150^0$ with x-axis and y-intercept is $-3$. $[\tan 150^0=-1/\sqrt{3},\quad x+\sqrt{3}y+3\sqrt{3}=0]$

**Examples on two-point form**

1) Find the equation of a line passing through the point $(6,-4),(-3,8)$. Also find the slope, x-intercept and y-intercept of the line. **[IoPE2015]** $[4x+3y-12=0,\ x-\text{intercept}=3,\ y-\text{intercept}=4]$

2) Find the equation of the line passing through the point of intersection of the lines 3x – 2y = 4 and 4x – 5y = 3 and through the point (3,2). **[IoPE2017]** $[(x,y)=(2,1)=(x_1,y_1),\quad x-y-1=0]$

**Examples on perpendicular and parallel lines**

1) Show that the lines $3x+2y=5$ and $2x-3y=6$ are perpendicular. **[BTE 2016]**

2) Find the value of $k$ if the lines $kx-6y-9=0$ and $6x+5y-13=0$ are perpendicular to each other. **[BTE 2017]** $[k=5]$

3) $2x+3y+7=0$ and $4x+6y+2=0$ are two straight lines. Are they parallel to each other? $[m_1=m_2=-2/3]$

4) Find the equation of the line passing through the point (4,5) and perpendicular to the line 7x – 5y = 420. **[IoPE2009]**

5) Find the equation of a straight line passing through the point of intersection of lines $2x+3y=13$ and $5x-y=7$ and perpendicular to the line $3x-y+7=0$. [**BTE 2017]** $[x+3y-11=0]$

6) Find the equation of a line passing through the point (3,- 4) and parallel to the line 5x -2y +3 = 0. **[IoPE2015]** $[5x-2y-23=0]$

7) Find the equation of the line passing through the point of intersection of x – 2y – 5 = 0 and x + 3y – 10 = 0 and parallel to 3x + 2y = 0. **[IoPE2002]**

**Examples on two-intercept form/double intercept form**

1) Find the equation of a line whose intercept on the X-axis is double that on the Y-axis and passing through the point (4,1). **[BTE 2017]** $[x+2y=6]$

2) Find the equation of a line passing through the point $(12,-4)$ and sum of the intercepts made by the line on the co-ordinate axes is 10. **[BTE 2015]** $[2x+3y=12]$

**Example on normal form**

1) Reduce the following equations in the normal form and find the values of p and $\alpha$.

a) $x-\sqrt{3}y-12=0,\ \ b)\sqrt{3}x-y+5=0,\ c)\,x+\sqrt{3}y=7$

$[a)\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{-\sqrt{3}/2}{1/2}=-\sqrt{3}=-\tan(60)=\tan(360-60)=\tan 300^0\ \therefore\alpha=300$

$b)\,\alpha=150^0,\ c)\,\alpha=60^0]$

2) Find the equation of a line whose perpendicular distance from origin is 5 and inclination of perpendicular is $\pi^c/4$. $[x+y=5\sqrt{2}]$

**Examples for Tutorial**

**Examples on slope and intercept of a line**

A) Find the slope of a line whose inclination is as follows:

1) $90^0$ [Not defined] 2) $105^0$ $\left[m=\frac{1+\sqrt{3}}{1-\sqrt{3}}\right]$ 3) $\frac{5\pi^c}{6}$ $\left[m=-\frac{1}{\sqrt{3}}\right]$

B) Find the slope of a line passing through the following points:

1) $(2,5)$ and $(4,-6)$ $\left[m=-\frac{11}{2}\right]$ 2) $(2,\sqrt{3})$ and $(3,2\sqrt{3})$ $[m=\sqrt{3}]$

3) whose x-intercept is -2 and y-intercept is 3. $[m=3/2]$

C) Find the value of K if the slope of a line passing passing through the following points:

1) $(2,5)$ and $(K,3)$ is 2 [K = 1] 2) $(3,-5)$ and $(K,-1)$ is $1/3$ $[K=15]$

D) Find the slope and intercepts of the following lines:

1) $y=3x-4$ $[m=3,\ x-\text{intercept}=4/3,\ y-\text{intercept}=-4]$

2) $\frac{x}{2}-\frac{y}{3}=\frac{1}{4}$ $[m=3/2,\ x-\text{intercept}=1/2,\ y-\text{intercept}=-3/4]$

E) Using slopes, find the value of K if the following points are collinear.

1) $(2,3),(-1,K),(5,8)$ $[K=-2]$ 2) $(-1,3),(8,K),(2,1)$ $[K=-3]$

**Examples on slope-point form of a line**

1) Find the equation of line passing through the point $(2,-3)$ and having slope $2/3$. Also, find its X and Y – intercepts. **[IoPE 2017]**
$[2x-3y-13=0,\ \ x-\text{intercept}=13/2,\ y-\text{intercept}=-13/3]$

2) Find the equation of line passing through the point $(-3,2)$ and having slope $5/2$.**[BTE2016]**
$[5x-2y+19=0]$

3) Find the equation of line passing through the point $(-1,1)$ and making an angle $(\pi/4)^c$ with the line $2x+3y=6$.**[BTE2017]** $[x-5y+6=0]$

**Examples on slope-intercept form of a line**

Find the equation of following lines:

1) Slope = 2, y-intercept =12 [y = 2x + 12 ]

2) Slope = 3, y-intercept = -5 [y = 3x - 5]

3) which makes an angle of $150^0$ with x-axis and y-intercept is $-3$.

[$\tan 150^0 = -1/\sqrt{3}$, $x+\sqrt{3}y+3\sqrt{3}=0$]

**Examples on two-point form of a line**

A) Find the equation of a straight passing through the points following points. Also, find its slope, x intercept and y intercept.

1) (5,-1) and (2,6) **[IoPE 2008]** 2) (-2,4) and (6,7) **[ IoPE 2014]**

3) $(-4,6)$ and $(8,-3)$ **[IoPE2016]** [$3x+4y-12=0$, Slope $=-\frac{3}{4}$, $x-$intercept $=4$, $y-$intercept $=3$]

B) 1) Find the equation of line passing through the point of intersection of the lines $x+y=0$ and $2x-y=9$ and through the point (2,5). [**BTE2016, 2017]** [$8x+y-21=0$]

2) Find the equation of line passing through the point of intersection of the lines $2x+3y=13$ and $5x-y=7$ and through the point $(1,-1)$. [**BTE2015]** [$4x-y-5=0$]

**Examples on perpendicular and parallel lines**

1) Show that the lines $2x+3y-1=0$ and $3x+2y+6=0$ are perpendicular. **[BTE 2016]**

2) Find the value of $p$ if the lines $3x+4py+8=0$ and $3py-9x+10=0$ are perpendicular to each other. **[BTE 2017]** $\left[p=\pm\frac{3}{2}\right]$

3) Show that the points $(6,1),(-1,8),(3,-2)$ are the vertices of right angled triangle by using slopes. **[BTE 2017]**

4) Find the value of K so that the line through the points $(2,7)$ and $(3,K)$ is parallel to the line through the points $(0,6)$ and $(-1,4)$. [$K=9$]

5) Find the equation of the straight line passing through the point (3,-4) and perpendicular to the line 7x – 5y + 3 = 0. **[IoPE2010]**

6) Find the equation of a straight line passing through the point of intersection of lines $2x+3y=13$ and $5x-y=7$ and perpendicular to the line $2x-5y+7=0$. **[BTE 2016**]

[$5x+3y-16=0$]

7) Find the equation of a straight line passing through the point of intersection of lines $2x+3y=13$ and $5x-y-7=0$ and perpendicular to the line $3x-2y+7=0$. **[BTE 2016]**

[$2x+3y-13=0$]

8) Find the equation of a line passing through the point (3, -2) and perpendicular to the line passing through (2, 9) and (-3, 6) **[IoPE2009]**

9) Find the equation of a line passing through the point (3,4) and parallel to the line 2x – 3y + 5 =0. **[IoPE2016]** [$2x-3y-18=0$]

10) Find the equation of a straight line passing through the point of intersection of lines $4x+3y=8$ and $x+y=1$ and parallel to the line $5x-7y=3$. [**BTE2017]** [$5x-7y-53=0$] **[**

**Examples on two-intercept form/double intercept form**

1) Find the equation of a line passing through the point (-3,8) and sum of the intercepts made by the line on the co-ordinate axes is 7. **[IoPE2017]** $[2x - y + 14 = 0$ and $4x + 3y - 12 = 0]$

2) Find the equation of the line passing through the point (-3,10) and the sum of x and y intercepts is 8. **[IoPE2012]**

3) Find the equation of a line making equal positive intercepts on the co-ordinate axes and passing through the point (-2,7). **[IoPE2016]** $[x + y = 5]$

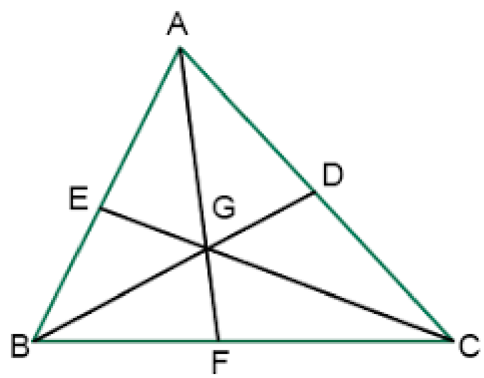
**Example on normal form of a line**

1) Reduce the following equations to the normal form:
(a) x + 5 = 0, (b) 2y – 3 = 0, (c) 4x + 3y – 9 = 0.

2) Find the equation of a line whose perpendicular distance from origin is 3 and inclination of perpendicular is $30^{0c}$. $[\sqrt{3}x + y = 6]$

**Perpendicular Bisector**

The perpendicular bisector of a side of a triangle is a line segment that is both perpendicular to a side of a triangle and passes through its midpoint.




In the fugure, seg.AF, BD and CE are perpendicular bisectors.

**Procedure to find the equation of perpendicular bisector of a line**

1) First find the slope of given points. say $m_1$.

2) Let slope of perpendicular line be $m_2$.

3) Find the slope of perpendicular line using the formula $m_1 m_2 = -1$.

4) Find the mid-point of given points.

5) Use slope-point formula by considering the slope of perpendicular line and the mid-point. The required line is the equation of perpendicular bisector.

**Examples**

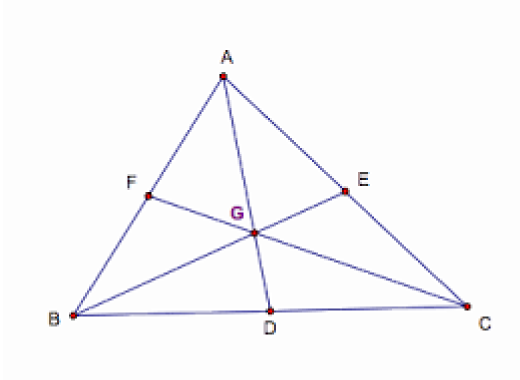
1) Find the equation of the perpendicular bisector of the line joining the points (8,1) and (2,-7). **[IoPE2012]**

2) Find the equation of straight line which is perpendicular bisector of the line joining the points $A(8,-1)$ and $B(6,3)$. **[BTE 2017]** $[x - 2y - 5 = 0]$

**Median of a triangle**

A median of a triangle is a line segment from a vertex of the triangle to the midpoint of the side opposite that vertex.

$\Delta ABC$ is a triangle as shown in the figure. There are three medians in a triangle. They are seg. AD, seg. BE and seg. CF as shown in the figure.




**Procedure to find equation of median of a triangle**

Suppose we want to find the equation of median AD.

1) Suppose D is the mid-point of seg. BC. Then find the co-ordinates of the point D by using mid-point formula.

2) Use two-point form by considering the points A and D.

3) The required line is equation of median AD.

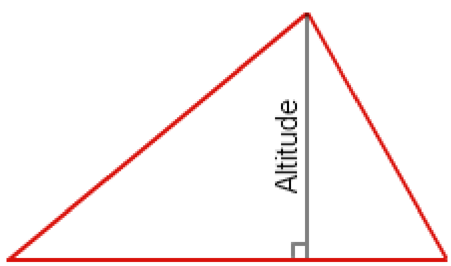
**Example**

1) If $A(3,1)$, $B(-1,3)$ and $C(=3,-2)$ are the vertices of $\Delta ABC$, find the equation of median AD.

$[x-10y+7=0]$

**Altitude of a triangle**

The altitude of a triangle is defined as a perpendicular drawn from any vertex (a point where two sides of a triangle meet) on to the opposite side (base)of that triangle.




There are three altitudes in a triangle.

**Procedure to find equation of altitude**

Suppose $\Delta ABC$ is a triangle. We want to find the equation of altitude from the point A.

1) Find the slope of seg. BC, say $m_1$.

2) Let $m_2$ be slope of seg. AD. It is altitude from the point A and perpendicular to seg. BC.

3) Use $m_1 m_2 = -1$, find the slope of seg. AD i.e. $m_2$.

4) Use slope-point formula to find the equation of altitude AD.

**Example**

1) If A(2,-5), B(-2,1) and C(4,7) are the vertices of $\Delta ABC$, find the equation of altitude from the point A. **[IoPE2017]** $[x+y+3=0]$

**Examples for Tutorial**

1) Find the equation of the perpendicular bisector of the line segment joining the points A(2,3) and B(6,-3). **[IoPE 2013]**

2) If A(2,5), B(6,-1) and C(-4,-3) are the vertices of a triangle. Find the equations of
(a) the median through the points A,B and C; ( b) the altitude through the points A,B and C;
(c) the perpendicular bisector through the points A,B and C.

## Perpendicular distance of a point from a line:

The perpendicular distance of a point $P(x_1, y_1)$ from the line $ax+by+c=0$ is denoted by $p$.

It is given by,

$$p=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|.$$

**Examples**

1) Find the length of perpendicular on the line $3x+4y-6=0$ from the point $(3,4)$. **[BTE 2015]**
$[p=19/5=3.8]$

2) Find the value of k for which the length of perpendicular from the point (4,1) to the line 3x – 4y + k = 0 is 2 units. **[IoPE2017] [ k = 2]**

## Perpendicular Distance between two parallel lines

The perpendicular distance between two parallel lines $ax+by+c=0$ and $ax+by+c'=0$ is given by,

$$d=\left|\frac{c-c'}{\sqrt{a^2+b^2}}\right|$$

**Example =** Find the perpendicular distance between two parallel lines

$3x+2y-6=0$ and $6x+4y-24=0$ **[BTE 2016]** $[p=\frac{6}{\sqrt{13}}]$

**Examples for Tutorial**

1) Find the length of perpendicular from the point $(2,3)$ on the line $4x-6y-3=0$. **[BTE 2015]**

$[p=\frac{\sqrt{13}}{2}]$

2) Find the length of perpendicular from the point (-3,- 4) on the line $4(x+2)=3(y-4)$.
**[BTE 2017]** **[ p= 4]**

3) Find the length of perpendicular from the point (-3,- 4) on the line $4x-3y+20=0$
**[BTE 2016]** **[p=4]**

4) Find the perpendicular distance between two parallel lines.

a) $3x+4y+5=0$ and $6x+8y=25$ $[7/2]$

b) $y=2x-4$ and $y=2x+3$ $[1/5]$

**Angle between two lines:** If $\theta$ is the acute angle between two lines having slope $m_1$ and $m_2$ then,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|.$$

**Note –** If $m_1 = m_2$, then $\tan\theta = 0$. So, the lines are parallel.

**Examples:**

Find the acute angle between the following lines:

1) $3x - 2y + 4 = 0$ and $2x - 3y - 7 = 0$ **[BTE2015, 2017]** $\left[\theta = \tan^{-1}\left(\frac{5}{12}\right) = \tan^{-1}(0.417)\right]$

2) $3x - y + 4 = 0$ and $2x + y - 3 = 0$ **[BTE 2017]** $\left[\theta = 45^0 \text{ or } \frac{\pi}{4}\right]$

3) x + 3y +5 = 0 and x – 2y = 4 **[IoPE2014,15]** $\left[\theta = 45^0 \text{ or } \frac{\pi}{4}\right]$

**Examples for Tutorial**

Find the acute angle between the following lines:

1) $y = 5x + 6$ and $y = x$ $\left[\theta = \tan^{-1}\left(\frac{2}{3}\right)\right]$

2) $3x - 4y = 420$ and $4x + 3y = 420$ $\left[\theta = 90^0 \text{ or } \frac{\pi^c}{2}\right]$

3) $x\cos\alpha + y\sin\alpha = p$ and $x\cos ec\alpha + y\sec\alpha = p$