

Logarithm

1) Define the following:

- a) Logarithm b) Common Logarithm c) Natural Logarithm
2) Write the relation between Common Logarithm and Natural Logarithm

3) Example: Find the value of the following by using Logarithm

Or Find the value of the following without using the calculator.

a) $\log_{2\sqrt{3}} 12$ [Answer : $x = 2$] b) $\log_{10} 1000$ [Answer : 3]

c) $\log_4 16$ [Answer : 2] d) $\log_5 0.20$ [Answer : -1]

4)

Example: Prove that

1) $\log(1+2+3) = \log 1 + \log 2 + \log 3$. 2) $\log\left(\frac{p}{q}\right) + \log\left(\frac{q}{r}\right) + \log\left(\frac{r}{p}\right) = 0$.

3) $\log 216 - \log 36 = \log 6$.

5) Write the values of following identities:

1) $\log_a 1 =$ 2) $\log_a a =$ 3) $a^{\log_a x} =$ 4) $\log_a a^x =$ 5) $\frac{1}{\log_a b} =$

6) Complete the following Laws of Logarithm.

1) $\log_a mn =$ 2) $\log_a (m/n) =$

3) $\log_a m^n =$ 4) $\log_b m =$

7) State True or False.

1) The base of a logarithm must be positive number other than 1.

2) Logarithm of negative numbers exist.

3) Logarithm of zero exist.

Partial Fraction

1) Define the following:

a) Rational fraction, b) Proper fraction, c) Improper fraction

d) Partial fraction

2) Resolve into partial fraction (Do not find out constants)

1) $\frac{1}{x(x+1)}$ 2) $\frac{3x-1}{(x-4)(2x+1)(x-1)}$ 3) $\frac{x-5}{x(x+3)(x-2)}$

4) $\frac{x^2}{(x+1)(x+2)^2}$ 5) $\frac{2x+3}{x^2(x-1)}$

6) $\frac{x^2+23x}{(x+3)(x^2+1)}$ 7) $\frac{x}{(x^3+1)}$ 8) $\frac{2x-3}{(x+1)(x^2+4)}$

Complex Numbers

- 1) Define complex number, conjugate of a complex number
- 2) If $z_1 = 4 - 5i$ and $z_2 = 3 + 7i$ then find $z_1 + z_2$, $z_1 - z_2$ and $z_1 \cdot z_2$.
- 3) If $z_1 = 5 - 2i$, $z_2 = 6 + 5i$, find $z_1 + z_2$, $z_1 - z_2$ and $z_1 \cdot z_2$.
- 4) Write the complex conjugates i and $2 - 3i$.

Determinant

- 1) Define Determinant.

2) Evaluate $\begin{vmatrix} 3 & -2 & 1 \\ 3 & -1 & -2 \\ 3 & -2 & -3 \end{vmatrix}$.

3) Solve $\begin{vmatrix} 2 & 3 & x \\ 1 & 0 & 3 \\ -2 & -1 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 8 \\ 2 & 1 \end{vmatrix}$.

- 4) Write Cramer's rule for three variables.

Matrices

- A) Define and write one example for the following:

- | | | |
|------------------------------------|---------------------------|------------------|
| 1) Matrix | 2) Row matrix | 3) Column matrix |
| 4) Square matrix | 5) Rectangular matrix | 6) Unit matrix |
| 7) Null matrix | 8) Diagonal matrix | 9) Scalar matrix |
| 10) Singular matrix | 11) Non-singular matrix | |
| 12) Determinant of a square matrix | 13) Transpose of a matrix | |

- 14) Adjoint of a matrix
- 15) Equality of two matrices

- B) 1) What is the order of a matrix if a matrix has 2 rows and 3 columns?

- C) Define multiplication of two matrices. When do you multiply two matrices?

- D) Define inverse of a matrix and write the formula to find inverse of a matrix.

- E) State true or false. Justify your answer.

- a) Matrices are denoted by small letters.
- b) A row matrix has many rows.
- c) A column matrix has only one column.
- d) Number of rows and columns are not equal in square matrix.
- e) Number of rows and columns are not equal in rectangular matrix.
- f) All elements of diagonal matrix are 0.
- g) All diagonal elements are equal in scalar matrix.
- h) Every scalar matrix is a unit matrix.
- i) The value of a determinant is 0 in singular matrix.
- j) The value of a determinant is 0 in non-singular matrix.
- k) $(A')' = A'$ i) $(A + B)' = A' - B'$ m) $(AB)' = B'A'$
- l) We add two matrices of different order.
- m) We subtract two matrices of same order.
- n) All elements of null matrix are 0.
- o) Diagonal elements of scalar matrix are not equal.

F) 1) Example: If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 7 \\ -5 & 6 \\ -4 & 4 \end{bmatrix}$, find A' and B' .

2) Example: If $A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, find A' and B' .

3) Example: If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 7 \\ -5 & 6 \\ -4 & 4 \end{bmatrix}$, can you multiply AB and $B'A'$?

4) Example: If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 7 \\ -5 & 6 \\ -4 & 4 \end{bmatrix}$, what are the orders of AB and $B'A'$?

5) If $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ -2 & 0 \end{bmatrix}$, then find $A + B$.

6) If $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ -2 & 0 \end{bmatrix}$, then find $A - B$.

7) If $A = \begin{bmatrix} 3 & 1 & -1 \\ 3 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 3 & -1 \end{bmatrix}$, what is the order of AB ?

8) If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$, find $|A|$ and $|B|$.

9) Given $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 7 \\ -5 & 6 \\ -4 & 4 \end{bmatrix}$, what are the order of A' and B' ?

10) $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$, find $A^T + B^T$ and $A^T - B^T$.

11) Find the minor and cofactor of elements in the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 3 & -2 \\ 5 & -1 & 0 \end{bmatrix}$.

Trigonometry

1) Define trigonometric ratios and write all trigonometric ratios.
Define compound angles and allied angles, multiple angles and sub-multiple angles.

Define factorization and defactorization and justify by an example.

Define inverse trigonometric functions and examples of such functions.

Define principal value of an inverse trigonometric functions

c) If $\sin A = 0.4$, find $\sin 3A$.

Example 1) If $\sin \theta = 0.6$, find $\sin 3\theta$.

2) If $\sin \theta = 0.6$, find $\sin 3\theta$. [$\sin 3\theta = 0.936$]

3) If $\sin \theta = 0.4$, find $\cos 3\theta$. [$\cos 3\theta = 0.3297$]

5) Without using the calculator, find the value of $\sin 75^\circ$

$\sin 15^\circ$ a) $\cos(75^\circ)$. b) $\tan(75^\circ)$ c) $\sin 210^\circ$ d) $\sec 3660^\circ$

Examples: Without using the calculator, find the value of

a) $\sin 420^\circ$ b) $\cos 390^\circ$ c) $\cos(-300^\circ)$ d) $\sin(-330^\circ)$ e) $\sec 3660^\circ$ f) $\sin 495^\circ$

A) Express the following as a sum or difference of trigonometric functions.

1) $2\sin 4\theta \cos 2\theta$ 2) $2\cos 117^\circ \sin 53^\circ$ 3) $2\cos 4\theta \cos 2\theta$ 4) $\sin(\theta/4)\sin(3\theta/4)$

1) If $2\cos 60^\circ \cdot \cos 10^\circ = \cos A + \cos B$, then find A and B.

8) Express the following as product of trigonometric functions:

a) $\sin 7\theta - \sin 5\theta$ b) $\sin 2\theta + \sin 4\theta$ c) $\cos 70^\circ + \cos 20^\circ$ d) $\sin 81^\circ - \sin 99^\circ$

e) $\cos \frac{\pi}{13} - \cos \frac{2\pi}{13}$

1) If $\sin 80^\circ + \sin 50^\circ = 2\sin \alpha \cdot \cos \beta$, find α and β .

Example: Find the principal value of the following :

1) $\tan^{-1}(-1)$ 2) $\sin^{-1}\left(-\frac{1}{2}\right)$ 3) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ 4) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ 5) $\cot^{-1} 1$

Straight Line

- A) 1) Define slope of a line. What are the slopes of x-axis and y-axis?
2) When do you say that the three points A,B, and C are collinear?
3) Write the equation of a line having slope m and passing through the origin.
4) Define x-intercept and y-intercept of a line. How to find x-intercept and y-intercept of a line?
5) Find the slope of a line whose inclination is as follows:

1) 90^0 2) 105^0 3) $\frac{5\pi^c}{6}$ 4) 45^0 5) 150^0 6) $\frac{2\pi^c}{3}$

- B) Find the slope of a line passing through the following points:

1) $(2, \sqrt{3})$ and $(3, 2\sqrt{3})$ 2) $(2, 5)$ and $(4, -6)$ 3) $(-1, -2)$ and $(-3, 8)$ 4) $(1, -3)$ and $(-1, -1)$

- C) Find the equation of a line passing through the points

1) $(-4, 6)$ and $(8, -3)$ 2) $(5, -1)$ and $(2, 6)$ 3) $(-2, 4)$ and $(6, 7)$ 4) $(6, -4), (-3, 8)$.

- D) Find the slope and intercepts of the following lines:

1) $3x + 4y = 12$ 2) $\frac{2x}{3} + \frac{y}{4} = 5$ 3) $y = 3x - 4$ 4) $\frac{x}{2} - \frac{y}{3} = \frac{1}{4}$

- E) Find the equation of a line

a) which makes an angle of 150^0 with x-axis and y-intercept is -3.

b) with a slope $1/2$ and y-intercept is 7.

c) slope = 2, y-intercept = 12.

d) slope = 3, y-intercept = -5.

- F)

- 1) Find the equation of a line having

a) x-intercept is -3 and y-intercept is 4. b) x-intercept is -2 and y-intercept is 3.

- G)

- 1) Find the equation of line passing through the point $(2, -3)$ and having slope $2/3$.

Also, find its x and y-intercepts.

- 2) Find the equation of a line passing through the point $(-3, 2)$ and having slope $5/2$.

- 3) Find the equation of line passing through the point $(-1, 1)$ and making an angle $(\pi/4)^c$ with the line $2x + 3y = 6$.

- 4) Find the equation of a line passing through the point $(2, -3)$ and making an angle

135^0 of with positive direction of X-axis. Also, find its X and Y-intercepts.

- H)

- 1) Find the equation of a line whose perpendicular distance from origin is 3

and inclination of perpendicular is 30^0 .

- 2) Find the equation of a line whose perpendicular distance from origin is 5

and inclination of perpendicular is $(\pi/4)^c$.

- 3) Find the equation of a line whose perpendicular distance from origin is 1

and inclination of perpendicular is 90^0 .

l)

- 1) When do you say that two lines are perpendicular to each other?
- 2) When do you say that two lines are parallel to each other?
- 3) Show that the lines $5x+6y-1=0$ and $6x-5y+3=0$ are perpendicular.
- 4) Show that the lines $3x+2y=5$ and $2x-3y=6$ are perpendicular.
- 5) Show that the lines $2x+3y-1=0$ and $3x+2y+6=0$ are perpendicular.
- 6) Find the value of k if the lines $kx-6y-9=0$ and $6x+5y-13=0$ are perpendicular to each other.
- 7) Find the value of p if the lines $3x+4py+8=0$ and $3py-9x+10=0$ are perpendicular to each other.
- 8) $2x+3y+7=0$ and $4x+6y+2=0$ are two straight lines. Are they parallel to each other?