

## Mensuration

**Definition of Mensuration** – Mensuration is the branch of mathematics which deals with the study of different geometrical shapes, their areas and volumes.

### Examples

1) The area of a rectangular courtyard is 3000 sq. m. Its sides are in the ratio 6:5. Find the perimeter of the courtyard. [ Perimeter = 60+60+50+50 = 220 m.]

2) Find the length of the longest pole that can be placed in a room 12 m long, 9 m broad and 8 m high.

$$[\text{Longest Pole} = \sqrt{l^2 + b^2 + h^2} = \sqrt{289} = 17m]$$

3) Find the area of ring between two concentric circles whose circumferences are 77 cm. and 55 cm.

$$[\text{Circumference of a circle} = 2\pi r, \text{ Area of a circle} = \pi r^2, \text{ Area of ring} = 231 \text{ sq.cm.}]$$

4) The diameter of a wheel is 28 cm. It rolls through a distance of 22 meters. How many revolutions will it make? [Hint: No. of revolution =  $\frac{\text{Distance through which wheel rolls}}{\text{Circumference of a wheel}} = 25$ ]

### Examples for Tutorial

1) Find the area between two concentric circles whose radii are 4m and 2m. [ $A = 37.7m^2$ ]

2) The diameter of a wheel is 28 cm. It rolls through a distance of 44m. Find the number of revolutions will it make? [50]

3) The area of a right angled triangle is 600 sq.cm. and one of the side containing right angle is 30 cm. Find the hypotenuse. [50cm.]

4) A rectangular park of length 30 m and breadth 24 m is surrounded by a 4 m wide path. Find the area of the path. [ $496m^2$ ]

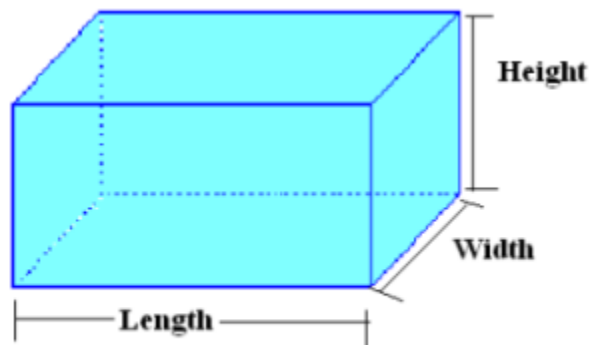
5) The length and breadth of a rectangle are in the ratio 3:2. If the area of the rectangle is  $726m^2$ , find its perimeter. [110 m]

### Mensuration of solid figures

1) **Parallelepiped** – A parallelepiped is a solid surface whose bases are parallelogram.

2) **Rectangular Parallelepiped or Cuboid** : A parallelepiped is said to be rectangular parallelepiped if all its faces are rectangles.

A cuboid is a three-dimensional shape with a length, width, and a height. The cuboid shape has six sides called faces. Each face of a cuboid is a rectangle.



**Cuboid**

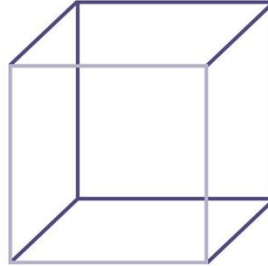
If  $l$ ,  $b$ ,  $h$  and  $d$  are the length, breadth, height and diagonal of a cuboid, then,

1) Surface area = Sum of areas of six rectangular faces =  $2[lb + bh + lh]$  sq. units

2) Diagonal of a cuboid =  $\sqrt{l^2 + b^2 + h^2}$  units

3) Volume of a cuboid =  $l \times b \times h$  cubic units

3) **Cube** – A rectangular parallelepiped in which each face is a square is called a cube.



**Cube**

If  $a$  is the side of a cube, then,

1) Surface area of a cube =  $2(a^2 + a^2 + a^2) = 6a^2$ , sq. units

2) Diagonal of a cube,  $d = \sqrt{a^2 + a^2 + a^2} = \sqrt{3}a$  units

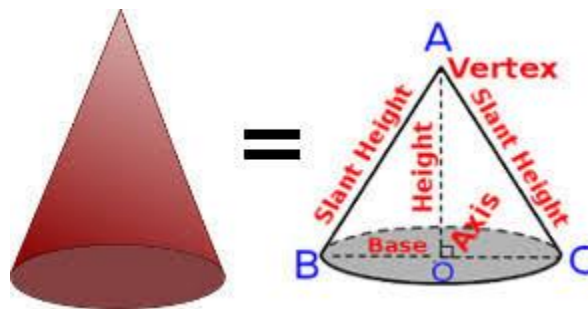
3) Volume of a cube,  $V = a \times a \times a = a^3$  cubic units

4) **Sphere** – If  $r$  is the radius of a sphere, then,

1) Surface area =  $4\pi r^2$  sq. units

2) Volume,  $V = \frac{4}{3}\pi r^3$  cubic units

5) **Right Circular Cone** - The lateral area of a right circular cone is equal to one-half the product of the circumference of the base  $c$  and the slant height  $L$ . The volume of the right circular cone is equal to one-third the product of the base area and the altitude.



**Right Circular Cone**

If  $r$  is the radius of the base,  $h$  is height of the cone and  $l$  is slant height of the cone then

1)  $r^2 + h^2 = l^2$

2) Area of base =  $\pi r^2$

3) Circumference of base =  $2\pi r$

4)  $S_c$  = Curved surface area

$$\begin{aligned} &= \frac{1}{2}(\text{Circumference of base}) \times \text{Slant height} \\ &= \frac{1}{2}(2\pi r) \times l = \pi r l = \pi r \sqrt{r^2 + h^2} \end{aligned}$$

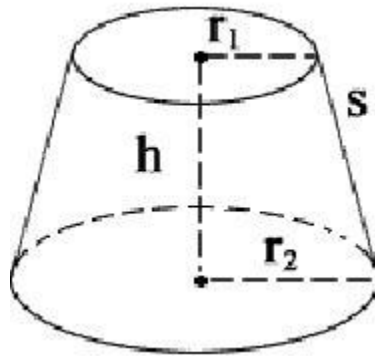
5)  $S_t$  = Total surface area

$$= \pi r l + \pi r^2 = \pi r(l + r)$$

6) Volume of cone =  $\frac{1}{3}(\text{Area of base}) \times \text{Height}$

$$\therefore V = \frac{1}{3} \pi r^2 h \text{ cubic units}$$

**6) Frustum of a cone** – Frustum of a cone is the solid obtained after removing the upper portion of it by a plane parallel to its base. The perpendicular distance between the parallel plane and base is called its thickness  $h$ .



#### Frustum of a cone

Let  $r_1$  be radius of circular top and  $r_2$  be radius of circular base ( $r_2 > r_1$ ),  $l$  is slant height, then,

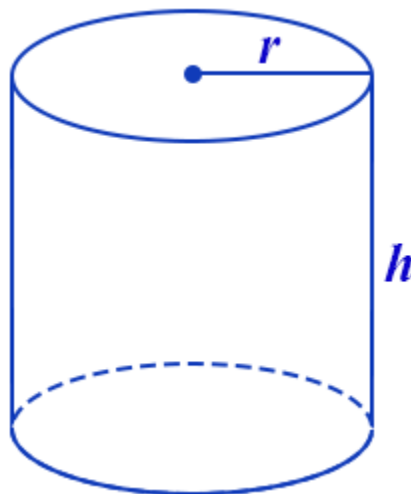
1) Volume of a frustum of a cone =  $\frac{h}{3}[\pi r_1^2 + \pi r_2^2 + \sqrt{\pi r_1^2 \cdot \pi r_2^2}]$

$$\therefore V = \frac{\pi h}{3}[r_1^2 + r_2^2 + r_1 r_2]$$

2) Curved surface of conical frustum =  $\pi(r_1 + r_2) \cdot l$  sq. units

3) Relation between slant height and perpendicular distance :  $l^2 = h^2 + (r_2 - r_1)^2$

**7) Cylinder** – When the number of sides of a prism is indefinitely increased, but the perimeter remains the same, then the solid is called a cylinder.



Cylinder

If  $h$  is the height and  $r$  is the radius of the base of the cylinder, then,

1)  $S_c = \text{Curved surface area} = 2\pi rh$ , sq. units

2)  $S_t = \text{Total surface area} = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$  sq. units

3) Volume,  $V = \pi r^2 h$ , cubic units

### Examples

1) If the volume of a cube is 42 c.c., find its surface area. [ $4\pi cm^2$ ]

2) Three metal cubes with edges 6cm., 8cm., and 10 cm., respectively are melted together and formed into a single cube. Find the diagonal of cube. [Diagonal =  $12\sqrt{3}$  cm.]

3) If the volume of a sphere is  $\frac{4\pi}{3} cm^3$ , find its surface area.

4) The slant height of a right circular cone is 10 m, and its height is 8 m, find the area of its curved surface. [ $S_c = 60\pi cm^2$ ]

5) The diameter of the base of a right circular cylinder is 42 cm. and its height 10 cm. Find the area of the curved surface and volume of the cylinder. [ $S_c = 2\pi RH = 1340 cm^2$ ,  $V = \pi R^2 H = 13860 cm^3$ ]

6) A lead bar 10 cm x 5 cm. x 4 cm. is melted and made into 5 equal spherical bullets. Find the diameter and surface area of the bullet. [  $D = 2r = 2(2.122) = 4.244$  cm.] [ $S.A. = 4\pi R^2 = 56.58 cm^2$ ]

7) A bucket is in the shape of a frustum of a cone whose radii of the top and the bottom are 15 cm. and 10 cm. respectively. If the depth of a bucket is 21 cm., calculate its capacity in litres. [  $V = 10.450$  litre]

### Examples for Tutorial

1) The edges of three cubes of metal are 3 dm, 4 dm. and 5 dm. They are melted and converted into a single cube. Find the edge of new cube. [  $a = 6$  dm.]

2) The volume of a sphere is  $\frac{88}{21} m^3$ . Find its surface area.

3) Find the volume of a right circular cylinder whose diameter is 6 cm. and slant height is 5 cm.  
[  $V = \frac{264}{7} cm^3$  ]

4) Find the curved surface area of a cone having diameter 2m and height  $4\sqrt{3}$  m. [ $7\pi m^2$ ]

5) A right circular cone has a slant height 3 times the radius of the base. If the area of the curved surface of the cone is  $18.48 cm^2$ , find the diameter of the base. [2.8 cm.]

6) If the whole area of the surface of a cylinder is  $1980 m^2$  and the perimeter of the base is 66 m, calculate its height. [  $h = 19.51$  m]