Matrices

Definition: An arrangement of mn numbers along m rows and n columns and bounded by the brackets $(\ )$ or $[\ ]$ is called an m by n matrix. It is denoted by $m \times n$ matrix.

Matrices are denoted by capital letters.

Types of matrices:

Row matrix: A matrix having only one row is called row matrix.

e.g. $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

Column matrix: A matrix having only one column is called column matrix.

e.g. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Square matrix: A matrix having equal number of rows and columns is called square matrix.

e.g. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Singular matrix: A square matrix A is called singular if its determinant is zero. i.e. $|A| = 0$.

Non-singular matrix: A square matrix A is called non-singular if its determinant is not zero. i.e. $|A| \neq 0$.

Diagonal matrix: A square matrix all of whose elements except those in the leading diagonal are zero is called diagonal matrix.

e.g. $\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ or $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

Scalar matrix: A square matrix in which all diagonal elements are equal is called scalar matrix.

e.g. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Unit or Identity matrix: A square matrix in which all diagonal elements are 1 is called unit or identity matrix. It is denoted by I.

e.g. A unit matrix of order 3 is $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Null or zero matrix: A matrix in which all elements are zero is called null or zero matrix. It is denoted by "**O**".

e.g. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Algebra of matrices:

1) Equality of matrices: Two matrices A and B are said to be equal if and only if
(a) they are of the same order.
(b) Each element of A is equal to the corresponding element of B.
2) Addition of matrices: If A and B are two matrices of the same order, then their sum A+B is a matrix in which every element is the sum of the corresponding elements of A and B.
3) Subtraction of matrices: If A and B are two matrices of the same order, then their difference A-B is a matrix which is obtained by subtracting the elements of B from the corresponding elements of A.
4) Scalar multiplication: The multiplication of a matrix A by a scalar (constant) k is a matrix whose every element is k times the corresponding elements of A.
5) Multiplication of matrices: Two matrices can be multiplied when the number of columns in the first matrix is equal to the number of rows in the second matrix. Such matrices are said to be conformable.
Transpose of a matrix: If A is a matrix, then a matrix obtained by interchanging rows and columns is called transpose of a matrix and it is denoted by A' or A^T.
Note: 1) $(A')' = A$, 2) $(A+B)' A' + B'$, 3) $(AB)' = B'A'$.
Adjoint of a square matrix: If A is a square matrix, then the adjoint of A is the transposed matrix of cofactors of A and it is written as AdjA.

$\therefore$ AdjA = Transposed matrix of cofactors of A

Inverse of a matrix: If A is a matrix then a matrix B such that AB = BA = I is called the inverse of A and it is denoted by A^{-1}.

$$\therefore A^{-1} = \frac{\text{Adj}A}{|A|} \text{ where} |A| \neq 0$$

Solution of Linear System of Equations: Consider the system of equations

$$a_1x + a_2y + a_3z = d_1$$
$$b_1x + b_2y + b_3z = d_2$$
$$c_1x + c_2y + c_3z = d_3$$

We write these equations in the matrix form AX = B--- (1)

$$\text{where } A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}.$$

A is called the coefficient matrix.

Examples on addition subtraction and scalar multiplication

1) If $A=\begin{bmatrix}1 & -2\\ 4 & 3\end{bmatrix}$, find the matrix X such that $A+2X=\begin{bmatrix}3 & 6\\ 0 & 1\end{bmatrix}$. **[BTE 2017]** $\left[X=\begin{bmatrix}1 & 4\\ -2 & -1\end{bmatrix}\right]$

2) If $A=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}4 & 5\\ 1 & -3\end{bmatrix}$, find the matrix X such that 2X + 3A - 2B=0. **[IoPE2015]**

$$\left\{X=\frac{1}{2}\begin{bmatrix}2 & 1\\ 5 & -18\end{bmatrix}\right\}$$

3) If $A=\begin{bmatrix}2 & 3\\ 4 & 7\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$, find $2A+3B-5I$ where I is the unit matrix of order two. **[BTE 2016]**

$$\left[2A+3B-5I=\begin{bmatrix}2 & 15\\ 2 & 24\end{bmatrix}\right]$$

Examples for Tutorial

1) If $A=\begin{bmatrix}2 & -1\\ 4 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & -2\\ -1 & 4\end{bmatrix}$, find the matrix X such that 2A+X = 3B. **[BTE 2017]**

$$\left\{X=\begin{bmatrix}5 & -4\\ -11 & 6\end{bmatrix}\right\}$$

2) If $A=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}4 & 5\\ 1 & -3\end{bmatrix}$, find the matrix X such that 2X + 3A - 2B=0. **[IoPE2015]**

$$\left\{X=\frac{1}{2}\begin{bmatrix}2 & 1\\ 5 & -18\end{bmatrix}\right\}$$

3) If $A=\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}$ find the matrix X such that $2X+3A-4B=I$ **[BTE 2016]**

$$\left\{X=\frac{1}{2}\begin{bmatrix}-4 & 11\\ -18 & -11\end{bmatrix}\right\}$$

4) If $A=\begin{bmatrix}2 & 3 & 1\\ 0 & -1 & 5\end{bmatrix}$, $B=\begin{bmatrix}1 & 2 & -6\\ 0 & -1 & 3\end{bmatrix}$, find $3A-B$. **[BTE 2016]** $\left\{3A-B=\begin{bmatrix}5 & 7 & 9\\ 0 & -2 & 12\end{bmatrix}\right\}$

5) If $A=\begin{bmatrix}2 & 3\\ 4 & 7\end{bmatrix}$, $B=\begin{bmatrix}1 & 3\\ 4 & 6\end{bmatrix}$, find $2A+3B-4I$, where I is the unit matrix of order two. **[BTE 2015]**

$$\left\{2A+3B-4I=\begin{bmatrix}3 & 15\\ 20 & 28\end{bmatrix}\right\}$$

6) If $A=\begin{bmatrix}4 & 1 & 5\\ -1 & 4 & -4\end{bmatrix}$, $B=\begin{bmatrix}3 & -3 & 0\\ 3 & 3 & 2\end{bmatrix}$, $C=\begin{bmatrix}1 & -1 & 4\\ 2 & 1 & 0\end{bmatrix}$, find $(A+B)+C$ and $4(A+B)$. **[IoPE.2016]**

$$\left\{Ans.\ (A+B)+C=\begin{bmatrix}8 & -3 & 9\\ 4 & 8 & -2\end{bmatrix}, 4(A+B)=\begin{bmatrix}28 & -8 & 20\\ 8 & 28 & -8\end{bmatrix}\right\}$$

Examples on determinant of a matrix

1) Verify that $|AB| = |A||B|$ where $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

Examples for Tutorial

1) If $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 3 \\ -1 & 5 \end{bmatrix}$, show that $|AB| = |A||B|$.

2) If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$, show that $|AB| = |A||B|$. Or find $|AB|$. **[BTE2017]** $[|AB| = -48]$

Examples on transpose of a matrix

1) If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$, find $A^T + B^T$ and $A^T - B^T$ **[BTE 2016]**

$$\left\{ A^T + B^T = \begin{bmatrix} 3 & 6 \\ -1 & 3 \end{bmatrix}, \quad A^T - B^T = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix} \right\}$$

2) If $A = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 6 \\ -3 & 4 \end{bmatrix}$, find $(AB)^T$. **[BTE 2016]**

$$\left\{ (AB)^T = \begin{bmatrix} -4 & 1 \\ 14 & 42 \end{bmatrix} \right\}$$

3) If $A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ find AB'. **[IoPE2016]**

$$\left\{ AB' = \begin{bmatrix} 2 & 7 & 0 \\ 1 & 2 & 12 \end{bmatrix} \right\}$$

Examples for Tutorial

1) If $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$, find $A^2 - 6A + 5I$, where I is the unit matrix. **[IoPE.2016]**

$$\left\{ A^2 = \begin{bmatrix} 13 & 12 \\ 12 & 13 \end{bmatrix}, A^2 - 6A + 8I = 0 \right\}$$

2) If $A = \begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 1 \\ 0 & 4 \\ 5 & 7 \end{bmatrix}$, find $(AB)^T$. **[BTE 2017]** $\left[(AB)^T = \begin{bmatrix} 42 & 10 \\ 64 & 18 \end{bmatrix} \right]$

3) If $A = \begin{bmatrix} 1 & 3 & 3 \\ 6 & 1 & 6 \\ -3 & -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \\ -1 & -1 & 0 \end{bmatrix}$, find $3A + 2B' + 3I$. **[IoPE 2017]**

$$\left\{ 3A + 2B' + 3I = \begin{bmatrix} 6 & 13 & 7 \\ 20 & 6 & 16 \\ -7 & -5 & 6 \end{bmatrix} \right\}$$

4) If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 7 \\ -5 & 6 \\ -4 & 4 \end{bmatrix}$, then show that $(AB)' = B'A'$. **[IoPE 2015]**

$$\left\{ (AB)' = B'A' = \begin{bmatrix} -17 & -19 \\ 28 & 23 \end{bmatrix} \right\}$$

Examples on multiplication of matrices

1) If $A=\begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$, show that A^2 is a null matrix.**[BTE2016, 2017]**

2) If $A=\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$, find (A - B) (A + B**). [IoPE2015]**

$\left\{(A-B)(A+B)=\begin{bmatrix} 10 & -20 \\ 20 & 10 \end{bmatrix}\right\}$

3) If $A=\begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 2 \end{bmatrix}, B=\begin{bmatrix} 1 & 3 \\ 3 & 0 \\ 0 & 1 \end{bmatrix}, C=\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ find $(AB)C$. **[BTE 2015]** $(AB)C=\begin{bmatrix} 23 & 4 \\ 5 & 17 \end{bmatrix}$

4) If $A=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that $A^2=\begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$. **[IoPE2017]**

Examples for Tutorial

1) If $A=\begin{bmatrix} 3 & 9 \\ -1 & -3 \end{bmatrix}$, show that A^2 is a null matrix.**[BTE 2017]**

2) If $A=\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$, prove that $A^2-3A=2I$, where I is the matrix of order two. **[BTE 2015]**

3) If $A=\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$, find $A^2-6A+5I$, where I is the unit matrix. **[IoPE.2016]**

$\left[A^2-6A+8I=0\right]$

4) If $A=\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find $A^2-9A+14I$ where I is the unit matrix. **[IoPE2017]**

$\left\{A^2=\begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}, A^2-9A+14I=0\right\}$

5) If $A=\begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$, find A^2.**. [IoPE 2016]** $\left[A^2=0\right]$

6) If $A=\begin{bmatrix} 3 & 1 & -1 \\ 3 & 1 & 2 \end{bmatrix}, B=\begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 3 & -1 \end{bmatrix}, C=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$, verify that $(AB)C=A(BC)$..**[BTE 2017]** $A(BC)=\begin{bmatrix} 14 \\ 14 \end{bmatrix}$

Examples on scalar matrices

1) If $A=\begin{bmatrix}2 & 4 & 4\\ 4 & 2 & 4\\ 4 & 4 & 2\end{bmatrix}$, show that A^2-8A is a scalar matrix. **.[BTE2015, 2017]**

Ans. $A^2-8A=\begin{bmatrix}20 & 0 & 0\\ 0 & 20 & 0\\ 0 & 0 & 20\end{bmatrix}$

2) Show that AB is a scalar matrix where

$A=\begin{bmatrix}3 & -2 & 0\\ 0 & -3 & 2\\ 2 & 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-3 & 2 & -2\\ 2 & 3 & -3\\ 3 & -2 & -9/2\end{bmatrix}$. **[IoPE2011]**

Examples for Tutorial

1) If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, show that A^2-4A is a scalar matrix. **[IoPE2015]**

Ans. $A^2=\begin{bmatrix}9 & 8 & 8\\ 8 & 9 & 8\\ 8 & 8 & 9\end{bmatrix}$, $A^2-4A=\begin{bmatrix}5 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 5\end{bmatrix}$

2) If $A=\begin{bmatrix}1 & 5 & 5\\ 5 & 1 & 5\\ 5 & 5 & 1\end{bmatrix}$, show that A^2-7A is a scalar matrix.

Examples on singular and non-singular matrices

1) If $A=\begin{bmatrix}-2 & 0 & 1\\ 1 & 2 & 3\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 2 & 3\\ 1 & 1\end{bmatrix}$ show that the matrix AB is non-singular. **[BTE 2015]** $\left[|AB|=17\neq 0\right]$

2) Find the value of k if the matrix $A=\begin{bmatrix}k-1 & 2 & 3\\ 3 & 1 & 2\\ 1 & -2 & 4\end{bmatrix}$ is singular. **.[IoPE015]** $[k=49/8]$

Examples for Tutorial

1) If $A=\begin{bmatrix}2 & -1\\ 4 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & -2\\ -1 & 4\end{bmatrix}$ whether the matrix $A-B$ is singular or non-singular? **[IoPE2016]**

$[|A-B|=-4\neq 0, \therefore |A-B| \text{ is non}-\text{singular matrix.}]$

2) If $A=\begin{bmatrix}2 & -1 & 3\\ 4 & 1 & -3\\ 0 & -1 & 1\end{bmatrix}$, find $|A|$ and verify that the matrix A is singular or non-singular matrix.

[BTE 2017]

$\left[|A|=-12\neq 0 \quad \therefore A \text{ is non-singular matrix.}\right]$

3) If $A=\begin{bmatrix}6 & -5 & 1\\ 4 & 2 & -1\\ 14 & -1 & k\end{bmatrix}$ is singular matrix, then find the values of k. **[IoPE2017]** $[k=-1]$

Examples on Equality of Matrices

1) If $\begin{bmatrix}3 & -6\\ 4 & 2\end{bmatrix}+\begin{bmatrix}2 & 3\\ -2 & 1\end{bmatrix}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, find the values of a, b, c ,d. [**BTE 2016**]

[a=5, b =-3, b = -3 , c = 2 , d =3]]

2) Find the values of x, y, z if $\begin{bmatrix}2+x & -1 & 3\\ 0 & y & z\\ 4 & 1 & 3\end{bmatrix}+\begin{bmatrix}1+x & 2 & 3\\ 0 & 1+y & 4\\ 2 & 3 & 5\end{bmatrix}=\begin{bmatrix}6 & 1 & 6\\ 0 & -1 & 6\\ 6 & 4 & 8\end{bmatrix}$. **[BTE 2015]**

$[x=3/2, \ y=-1, \ z=2]$

3) If $A=\begin{bmatrix}1 & 3 & 2\\ 3 & 0 & 1\\ 3 & 1 & 2\end{bmatrix}$, $B=\begin{bmatrix}3 & 0 & 2\\ 1 & 4 & 5\\ 2 & 1 & 0\end{bmatrix}$, $C=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$ and $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ such that $(A+2B)C=X$, find the values of x, y, z . **[BTE 2017]** $[x=31 \quad y=54 \quad z=19]$

Examples for Tutorial

1) If $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, find the values of x and y..**[IoPE2015]** $[x = 3 - 2 = 1]$

2) Find the values of x and y if $\left\{3\begin{bmatrix} 4 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} - 2\begin{bmatrix} 3 & 2 & 4 \\ -6 & 1 & -3 \end{bmatrix}\right\}\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$.**[IoPE2015]**

$[x = 1,\ y = 3]$

3) If $\begin{bmatrix} x+y & y-z \\ z-2x & y-x \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$, find the values of x, y and z. **[IoPE 2016]** $[x = 1,\ y = 2,\ z = 3]$

4) Find the values of x,y,z and a which satisfy the matrix equation $\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$.
[IoPE2016] $[x = -3,\ y = -2,\ z = 4,\ a = 3]$

5) Find the values of x, y, z if $\left\{\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 3 & 0 & 2 \\ 1 & 4 & 5 \\ 2 & 1 & 0 \end{bmatrix}\right\}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.**[BTE2015,2017]**

$[x = 31 \quad y = 53 \quad z = 19]$

Examples on inverse of a matrix

Find the inverse of following matrices by adjoint method where

1) $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$ $\qquad A^{-1} = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

2) $A = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 7 & 1 \\ 3 & 5 & -1 \end{bmatrix}$ $\qquad A^{-1} = \frac{1}{12}\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 3 & 2 & -4 \end{bmatrix}$

3) $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ **[BTE 2015,2017]** $\qquad A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Examples for Tutorial

Find the inverse of following matrices by adjoint method where

1) $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$ **[BTE2016, 2017]** $\quad A^{-1}=\begin{bmatrix}1 & -3 & 2\\ -3 & 3 & -1\\ 2 & -1 & 0\end{bmatrix}$

2) $A=\begin{bmatrix}2 & -1 & 0\\ 1 & 0 & 4\\ 1 & -1 & 1\end{bmatrix}$ **[BTE 2017]** $\quad A^{-1}=\frac{1}{5}\begin{bmatrix}4 & 1 & -4\\ 3 & 2 & -8\\ -1 & 1 & 1\end{bmatrix}$

3) $A=\begin{bmatrix}1 & 2 & 4\\ -1 & 2 & 3\\ 1 & 4 & 1\end{bmatrix}$ **[BTE 2016]** $\quad A^{-1}=\frac{1}{-26}\begin{bmatrix}-10 & 14 & -2\\ 4 & -3 & -7\\ -6 & -2 & 4\end{bmatrix}$

4) $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ **[BTE 2015]** $\quad A^{-1}=\begin{bmatrix}1 & -1 & 0\\ -2 & 3 & -4\\ -2 & 3 & -3\end{bmatrix}$

5) $A=\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$ **[IoPE2017]** $\quad A^{-1}=\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ -1 & 1 & 1\end{bmatrix}$

6) $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$.**[IoPE 2016]** $\quad A^{-1}=\begin{bmatrix}1 & -3 & 0\\ -2 & 3 & -4\\ -2 & 3 & -3\end{bmatrix}$

Examples on solution of linear equations by adjoint method

Solve the following equations by adjoint method.

1) $x-2y+z=0,\ 2x+3y-2z=2,\ 5x-y=3$ **[IoPE2017]** $\quad [x=1,\ y=2,\ z=3]$

2) $2x+y=3,\ 2y+3z=4,\ 2z+2x=8$ **[BTE 2017]** $\quad [x=2, y=-1, z=2]$

Examples for Tutorial

1) $x+3y+2z=6,\ 3x-2y+5z=5,\ 2x-3y+6z=7$ **[BTE 2015]** $\quad [x=-1, y=1, z=2]$
2) $2x+3y-z=-3,\ 5x+y+3z=10,\ 4x+3y-2z=-3$ **[BTE 2017]** $\quad [x=1, y=-1, z=2]$
3) $x+3y+3z=12,\ x+4y+4z=15,\ x+3y+4z=13$ **[BTE 2016]** $\quad [x=3, y=2, z=1]$
4) $x+y+z=3,\ 3x-2y+3z=4,\ 5x+5y+z=11$ **[BTE 2016]** $\quad [x=1, y=1, z=1]$
5) $x+y+z=3,\ x+2y+3z=4,\ x+4y+9z=6$ **[BTE 2017]** $\quad [x=2, y=1, z=0]$