Logarithm

In this chapter, we learn concept and laws of logarithm and solve examples on laws of logarithm.
Definition – The logarithm of a number m to the base a $(>0 \text{ and } \neq 1)$ is the index to which a must be raised x to get m, where x is any real number and a, m are positive real numbers.

$\therefore$ If $a^x = m$, then $\log_a m = x$.

Note:1) The base a must be positive number other than 1.
2) Logarithms of negative numbers and zero do not exist.
3) The common logarithm has 10 as its base.
4) The natural logarithm has the number "e" as its base.
5) Relation between common and natural logarithm : If N is a positive number then
$\log_e N = 2.303 \log_{10} N$

Important Identities:

1) $\log_a 1 = 0$ 2) $\log_a a = 1$ 3) $x = a^{\log_a x}$ 4) $x = \log_a a^x$ 5) $\log_b a = \frac{1}{\log_a b}$

Examples: Find the values of the following: 1) $\log_{10} 1000$ 2) $\log_{2\sqrt{3}} 12$

Examples for Tutorial

1) $\log_4 16$ [2] 2) $\log_5 0.20$ [−1]

Example: If $y = \log_3 (\log_2 x)$, find y when $x = 2$ and x when y = 1.

Example: If $y = \log_2 (\log_3 x)$, find y when $x = 3$ and x when y = 2.

Laws of logarithm:

1) $\log_a mn = \log_a m + \log_a n$ 2) $\log_a (m/n) = \log_a m - \log_a n$

3) $\log_a m^n = n.\log_a m$ 4) $\log_b m = \frac{\log_a m}{\log_a b}$ where a is a new base.

Examples: Find 1) $\frac{\log_5 243}{\log_{25} 9}$ 2) $\frac{\log_n m^2}{\log_{n^2} m}$

Examples: Prove that

1) $\frac{1}{\log_3 6} + \frac{1}{\log_8 6} + \frac{1}{\log_9 6} = 3$ [B.T.E.'89] 2) $\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc} = 3$[B.T.E.'85]

3) $\log(1+2+3) = \log 1 + \log 2 + \log 3$ 4) $\log\left(\frac{p}{q}\right) + \log\left(\frac{q}{r}\right) + \log\left(\frac{r}{p}\right) = 0$

Examples for Tutorial

Solve the following equations:

1) $\log_3 x + \log_3 4 = 2$ $[x = 9/4]$ 2) $\log_2\left(x^2 + 7\right) = 4$ $[x = \pm 3]$ 3) $e^{\sin x} = 1$ $[x = 0]$

4) $4^x + 1 = 2^{x+1}$ $[x = 0]$ 5) $\log_2\left[\log_2(\log_3 x)\right] = 1$ $[x = 81]$