Academic Year : 2018-19

Sem. II (All Courses) Subject – Applied Mathematics

Syllabus

Unit I - Function, Limit, Derivative
Unit II – Integration
Unit III - Definite Integral
Unit IV – Differential Equations
Unit V – Laplace Transform
Unit VI –Probability Distribution

Learning Resources/Reference Books –

1) Mathematics for polytechnic – Prof.S.P.Deshpande
2) A Textbook of Engineering Mathematics – Prof. G.V. Kumbhojkar
3) R.K.s Most Likely Question-papers Solution – Dr.R.K.Jadhav

E-learning Platform –

1) Website -

www.mathsrk.weebly.com

2) Mobile Apps in the form of E-Book:

Applied Mathematics,MathsRK2, MathRK 3-4,

Above mobile apps are available free on the website **www.amazon.com**

Important features of website and mobile apps -

- 2000 important formulae from Std, VII up to Ph,D, level
- Theory, solved and practice examples of Engineering Mathematics, Basic Mathematics and Applied Mathematics
- Previous question-papers of IoPE, Lonere and MSBTE
- Useful for all students and teachers of Science and Technology

Procedure to download apps:

1) First download **Amazon App store** in mobile app.
2) Create your account on the website www.amazon.com
3) Type the name of apps; ***MathsRK2*** **,** ***MathRK 3-4, Applied Mathematics*** in the search box and enter.
4) Then you see the app icon and click on *Get* and download.
5) After downloading, click on *install.*

Function

Function: If A and B are two non-empty sets, then a relation f from a set A to a set B is said to be function if every element of set A has one and only one image in set B. It is denoted by $f : A \to B$.

If f is a function from a set A to a set B and $(x, y) \in f$, then y = f(x). Here y is called image of x under f and x is called pre-image of y under f.

Value of a function: If y = f(x) is a function, then the value f(c) is obtained by putting x by c in the given function is called value of a function.

e.g. 1) If $f(x) = x^2 - 4x$, then $f(2) = (2)^2 - 4(2) = 4 - 8 = -4$.

2) If $f(x) = \log x$, then $f(3) = \log 3$.

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Types of functions

1) Even function: A function f(x) which does not change in sign when x changes its sign i.e. f(-x) = f(x) for all x is called an even function. **[R.E. Dec.2009, F.T.E.Aug.2010]**

e.g. $x^2, \cos x$ are even functions.

2) Odd function: A function f(x) which changes in sign when x changes its sign i.e. f(-x) = - f(x) for all x is called an odd function. **[F.T.E.Aug.2010]**

e.g. $x^3, \sin x$ are odd functions.

3) Composite function: Let $f : A \to B$ and $g : B \to C$ be two functions. Then the composite of f and g, denoted by gof , is defined as $gof : A \to C$ and given by,

$$gof(x) = g[f(x)], \text{ for all } x \in A.$$

The composite of g and f is denoted by fog. i.e. f[g(x)] is a composite function. So, function of a function is also called as composite function.

e.g. Let $A = \{1, 2, 3, 4, 5\}, B = \{1, 4, 9, 16\}$ and $C = \{a, b, c, d, e\}$ be three non-empty sets.

f = {(1,1), (2,4), (3,9), (4,16)} and g = {(1,a), (4,b), (9,c)} are two functions

4) Implicit function: A function of the form f(x, y) = 0 is called an implicit function.

Here there is a mixing of the variables x and y. They are not separated from each other.

e.g. $x^2 - 4xy + y^2 = 0, y = e^{xy}, \cos(x + y) = y^x, x + y = \log xy$ are some implicit functions.

5) Parametric function: A functional relation between x and y in terms of third variable is known as a parametric function.

It is more convenient to express x and y in terms of third variable θ (or t) , where θ (or t) is called parameter and x and y are parametric functions. In general, x = f(t) and y = g(t) are parametric functions in terms of parameter t.

e.g. $x = r\cos\theta$ and $y = r\sin\theta$ are parametric equations of circle.

6) Inverse function: If $f : A \to B$ is a function and we can find a function $g : B \to A$, then the function g is called an inverse function of f and it is denoted by $g = f^{-1}$.

i.e. if y = f(x) is a function, then $x = f^{-1}(y)$ is called an inverse function.

e.g. $y = \sin x$ and $x = \sin^{-1} y$ are inverse trigonometric functions of each other.

Examples on value of a function

1) If $f(x)$ is defined by $f(x) = 3x + 2$ when $-1 \le x \le 1$
$= x^2 - 1$ when $1 < x \le 2$
$= 2x - 5$ when $2 < x \le 3$.
Find the values of f(x) when x = -1,0,1.5 and 3. **[IoPE 2008]**

2) If $f(x) = (125)^x + 5\log_3 x$, find f(1/3). **[B.T.E.'90]**

3) If $f(x) = ax^2 + bx + 2$ and $f(1) = 3$, $f(4) = 42$, find the values of a and b. **[B.T.E.2017]**

4) If $f(t) = 50\sin(100\pi t + 0.04)$, show that $f\left(\frac{2}{100} + t\right) = f(t)$. **[B.T.E. 2015]**

5) If $f(x) = \tan x$ find $f(x + y)$ and $f(2x)$. **[[IoPE 2009,B.T.E.2015]**

Examples for Tutorial

1) If $f(x) = x^3 - x^2 + x - 1$, find $f(x) + f(-x)$. **[B.T.E.2016]** $\left[f(x) = -2x^2 - 2 \right]$

2) If $f(x) = 16^x + \log_2 x$, then find the value of $f\left(\frac{1}{4}\right)^2$ and $f\left(\frac{1}{2}\right)$. **[B.T.E.2016]**

$\left[f\left(\frac{1}{4}\right)^2 = 0 \text{ and } f\left(\frac{1}{2}\right) = 3 \right]$

3) Find the value of a if $f(x) = ax + 10$ and $f(1) = 13$. **[B.T.E.2017]** $\left[a = 3 \right]$

4) If $f(x) = x^2 - 3x + 4$, then solve $f(x) = f(2x + 1)$. **[B.T.E.2015,16,17]** $[x = 2/3 \text{ or } x = -1]$

5) If $f(t) = 4\sin(\omega t - 3P) - \cos\left(\omega t + \frac{5P}{3}\right)$, show that $f\left(\frac{P}{w} + t\right) = 4\sin wt - \cos 3wt$ if P = 0.

6) If $f(x) = \log\left(\frac{x+1}{x-1}\right)$, then prove that $f\left(\frac{1+x^2}{2x}\right) = 2f(x)$ **[B.T.E. 2015]**

Examples on even and odd functions

1) Determine whether the following functions are even or odd.
$(i)\ f(x) = 3x^4 + x^2 + 5 - 3\cos x + 2\sin^2 x$ $ii)\ f(x) = a^x - a^{-x}$ $iii)\ f(x) = \log(x + \sqrt{x^2 + 1})$ **[B.T.E.'86]**

Examples for Tutorial

1) Determine whether the following functions are even or odd.
i) $f(x) = x^7 - 5x^5 + 3\sin x$ **[IoPE 2017] [Odd]** $(ii)\ f(x) = x^4 - 5x^2 + x\sin x$ **[Even]**
$(iii)\ f(x) = \sin^3 x$ **[Odd]** iv) $f(x) = \sin(x^4 + 1)$ **[IoPE 2015] [Even]**
2) If $f(x) = x^3 - x^2 + x - 1$, find $f(x) + f(-x)$. **[B.T.E.2016]**

Examples on composite function

1) If $f=\{(2,4),(3,6),(4,8),(5,10),(6,12)\}, g=\{(4,13),(6,19),(8,25),(10,31),(12,37)\}$ find gof. **[IoPE 2016]**

2) If $f(x)=\sin^{-1}x$ and $\phi(x)=\sin x$, find $f[\phi(x)]$ and $\phi[f(x)]$. **[IoPE 2016, B.T.E.'91]**

3) If f(x) = 5x + 4 and g(x) = 4x + k and if $fog = gof$, then find the value of k. **[IoPE 2016]**

4) If $f(x)=\frac{1-x}{1+x}$, show that $f\{f[f(x)]\}=f(x)$. **[B.T.E.'May 92]**

Examples for Tutorial

1) If $f(x)=\frac{7-2x}{3x+2}$, show that $f[f(x)]=x$. **[IoPE 2007]**

2) If $f(x)=\frac{3x+4}{5x-7}$ and $g(x)=\frac{7x+4}{5x-3}$, show that $f[g(x)]=g[f(x)]$.

3) Find $f[g(x)]$ and $g[f(x)]$ if $(i)\,f(x)=3x-1$ and $g(x)=x^2+1$, $(ii)\,f(x)=2x$ and $g(x)=x^2$

$[(i)\,f[g(x)]=3x^2+2,\ g[f(x)]=9x^2-6x+2 \quad (ii)\,f[g(x)]=2x^2,\ g[f(x)]=4x^2]$

Examples on inverse function

1) If $y=f(x)=x^2+3x+4$, find $x=f^{-1}(y)$ and its domain. **[B.T.E.'86]**

2) If $f(x)=\frac{3x+2}{4x-3}$, show that $f=f^{-1}$. **[B.T.E.2015]**

Examples for Tutorial

1) If $f(x)=\frac{3x-4}{2x-3}$, show that $f^{-1}=f$.

2) If $y=f(x)=x^2+4x+1$, find $x=f^{-1}(y)$ . . $[x=-2\pm\sqrt{y+3}]$

3) If $y=f(x)=4x^2+4x+9$, find $x=f^{-1}(y)$. $\left[x=-\frac{1}{2}\pm\frac{\sqrt{y-8}}{2}\right]$

Examples on the type $x=f(t)$

1) Find $f(t)$ if $f(x)=\frac{2x+5}{3x-4}$ and $t=\frac{5+4x}{3x-2}$. **[B.T.E.2015]**

2) If $f(x)=\frac{x+5}{3x-4}$, $x\neq 4/3$ and $t=\frac{5+4x}{3x-1}$, find $f(t)$. **[B.T.E..'83]**

Examples for Tutorial

1) If $f(x)=\frac{x+2}{4x-3}$ and $t=\frac{2+3x}{4x-1}$, then show that $f(t)=x$.**[B.T.E.'91]**

2) If $f(x)=\frac{x+3}{4x-5}$ and $t=\frac{3+5x}{4x-1}$, find $f(t)$. **[B.T.E. 2015,16]** $\left[f(t)=x\right]$

3) If $f(x)=\frac{ax+b}{cx-d}$ and $t=\frac{b+dx}{cx-a}$, find $f(t)$. $\left[f(t)=x\right]$

Examples on the type $x = f(y)$

1) If $y = f(x) = \frac{2x-3}{3x-2}$, show that $f(y) = x$. **[IoPE 2011, B.T.E. 2016]**

2) If $y = f(x) = \frac{x-1}{x+1}$, show that $f(y) = -\frac{1}{x}$. **[B.T.E.'79]**

Examples for Tutorial

1) If $y = f(x) = \frac{3x-2}{2x-3}$, then show that $x = f(y)$. **[B.T.E.'92]**

2) If $y = f(x) = \frac{ax-b}{bx-a}$, , then show that $x = f(y)$.