Determinants

Definition of Determinant – An arrangement of numbers in equal number of rows and columns is called determinant and enclosed between two vertical bars is called determinant.

Determinant of order three: A determinant of order three is a square arrangement of 9 numbers a, b, c d, e, f; and g, h, i arranged in three rows and three columns enclosed between two vertical bars.

$$\therefore D = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

Cramer’s Rule for three variables: Let ax + by + cz = p , dx + ey + fz = q gx + hy + iz = r

and cx + dy = f be three linear equations in three variables x,y, z then their solutions are given by,

$$x = \frac{D_x}{D},\ y = \frac{D_y}{D}, z = \frac{D_z}{D} \text{ provided } D \neq 0$$

$$\text{where } D = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix},\ D_x = \begin{vmatrix} p & b & c \\ q & e & f \\ r & h & i \end{vmatrix}, D_y = \begin{vmatrix} a & p & c \\ d & q & f \\ g & r & i \end{vmatrix}, D_z = \begin{vmatrix} a & b & p \\ d & e & q \\ g & h & r \end{vmatrix}$$

Minor and cofactor of elements in a determinant: Let a_{ij} be element in the i^{th} row and j^{th} column of a determinant.

The minor of a_{ij} is defined as the value of the determinant obtained by eliminating the i^{th} row and j^{th} column of a determinant. It is denoted by M_{ij}.

The cofactor of a_{ij} is defined as the value of minor of a_{ij} with proper sign.

$$\therefore \text{Cofactor of } a_{ij} = (-1)^{i+j} (\text{Minor of } a_{ij})$$

Example

Evaluate the following determinants:

1) $\begin{vmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -5 & -6 & 0 \end{vmatrix}$ **[BTE2015]**

Examples for Tutorial

1) $\begin{vmatrix} 2 & 4 & 1 \\ 1 & 0 & 4 \\ 5 & -1 & 3 \end{vmatrix}$ **[75]**

2) $\begin{vmatrix} 3 & -2 & 1 \\ 3 & -1 & -2 \\ 3 & -2 & -3 \end{vmatrix}$ **[-12]**

Examples :

Solve the following determinants if

) $\begin{vmatrix} x & 2 & 1 \\ 3 & x & -2 \\ 1 & 3 & 1 \end{vmatrix} = 5$ **[IoPE2015,17]** \`   2) $\begin{vmatrix} x+2 & 1 & -3 \\ 1 & x-3 & -2 \\ -3 & -2 & 1 \end{vmatrix} = 0$ **[IoPE 2016]**

3) $\begin{vmatrix} 1 & 2 & 4 \\ 1 & x & x^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix}$ **[IoPE2017]**   4) $\begin{vmatrix} p & 4 & -4 \\ 3 & -2 & 1 \\ -2 & -4 & 1 \end{vmatrix} = 0$ **[BTE 2017]**

Examples for Tutorial

1) $\begin{vmatrix} -2 & 0 & 0 \\ -6 & x & 1 \\ -4 & 0 & -1 \end{vmatrix} = -4$ **[BTE 2016]** $[x = -2]$ 2) $\begin{vmatrix} 4 & 3 & 9 \\ 3 & 2 & 7 \\ 1 & 4 & x \end{vmatrix} = 0.$ **[BTE 2016]** $[x = -1]$

3) $\begin{vmatrix} 1 & 1 & 1 \\ 3 & x & 3 \\ 1 & x & 2 \end{vmatrix} = 0.$ **[BTE 2017]** $[x = 3]$ 4) $\begin{vmatrix} 0 & 7 & -2 \\ 11 & x & 10 \\ 4 & 8 & 1 \end{vmatrix} = 0.$ **[BTE 2017]** $\left[x = -\frac{27}{8} = -3.375 \right]$

5) $\begin{vmatrix} 2 & 3 & x \\ 1 & 0 & 3 \\ -2 & -1 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 8 \\ 2 & 1 \end{vmatrix}$ **[BTE 2017]** $[x = 5]$ 6) $\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 5 & 4 \\ 5 & 4 \end{vmatrix}$ **[BTE 2015]**

$[x = 1 \text{ or } x = 2]$

Examples:

Solve the following equations by using Cramer's rule of determinants.

1) $3x + 3y - z = 11,\ 2x - y + 2z = 9,\ 4x + 3y + 2z = 25$ **[BTE 2017]** $[x = 2,\ y = 3,\ z = 4]$

2) $x + y + z = 6,\ 2x - y + 3z = 9,\ x + 2y + 3z = 14$ **[BTE 2015]** $[x = 1,\ y = 2,\ z = 3]$

Examples for Tutorial

1) $x + y - z = 0,\ 2x + y + 3z = 9,\ x - y + z = 2$ **[BTE 2015]** $[x = 1,\ y = 1,\ z = 2]$

2) $x + y = 3,\ y + z = 5,\ x + z = 4$ **[BTE 2016]** $[x = 1,\ y = 2,\ z = 3]$

3) $x + y + z = 3,\ x - y + z = 1,\ x + y - 2z = 0$ $[x = y = z = 1]$ **[BTE 2017]**

4) $x + y = 5,\ y + z = 8,\ z + x = 7$ **[BTE 2017]** $[x = 2,\ y = 3,\ z = 5]$

5) $2x + 3y = 5,\ y - 3z = -2,\ z + 3x = 4$ **[IoPE16, BTE 2016]**